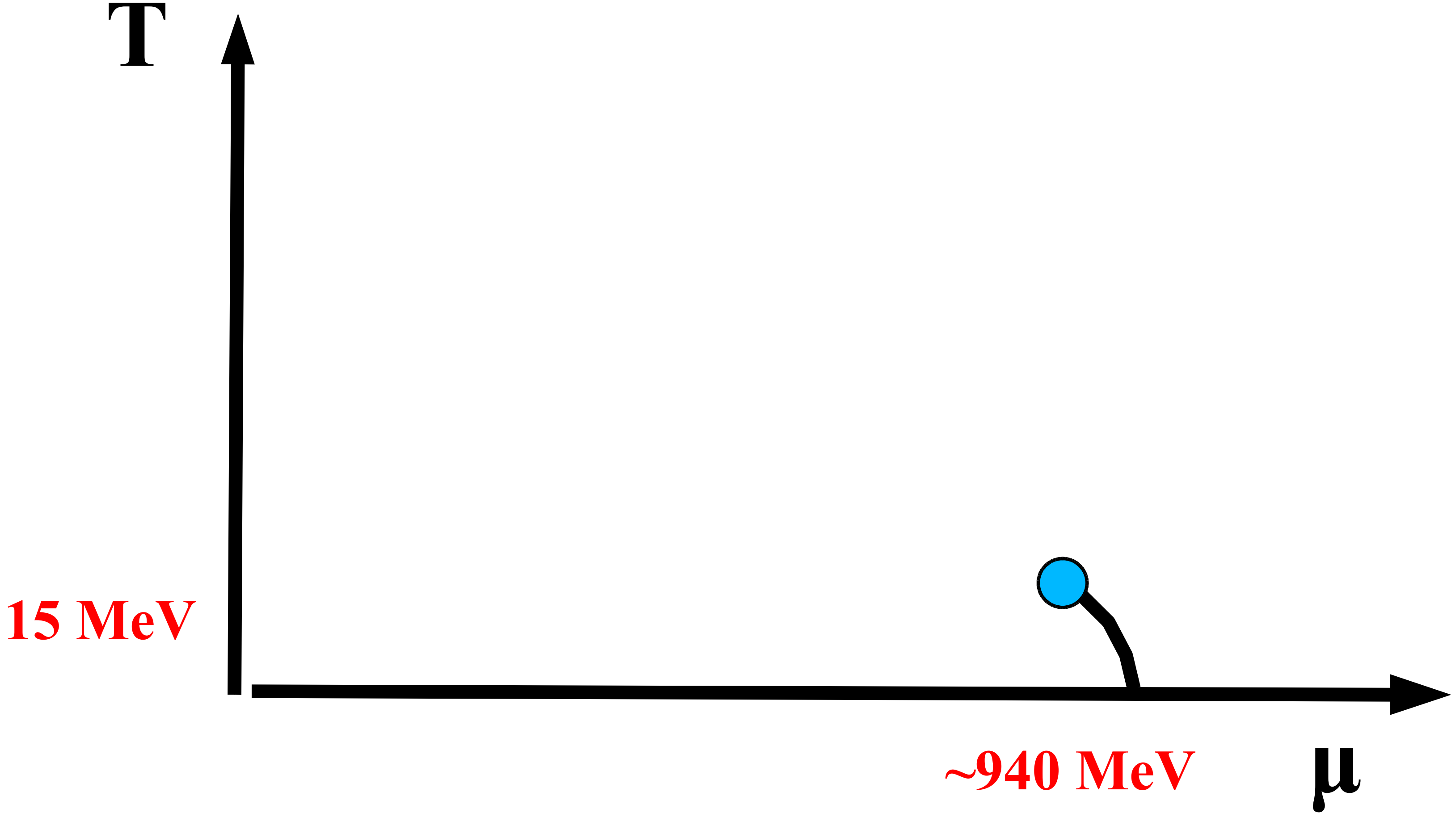
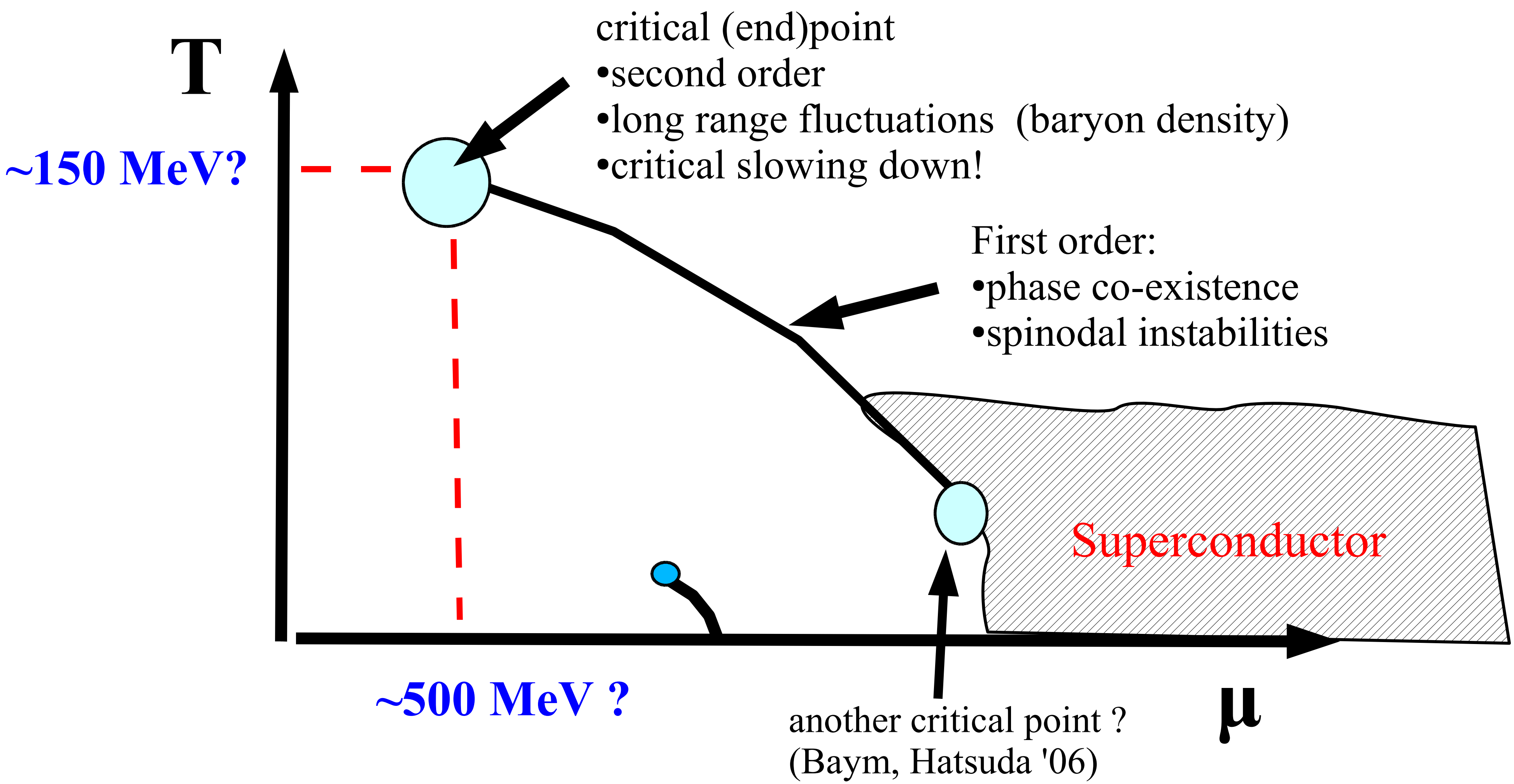
Exploring the Phasediagram Fluctuations and Correlations

- Introduction
- “Historical” perspective: Nuclear Liquid-Gas transition
- The QCD phase diagram
- Fluctuations, Correlations: Promises and Pitfalls
- Summary

The QCD Phase Diagram from experiment



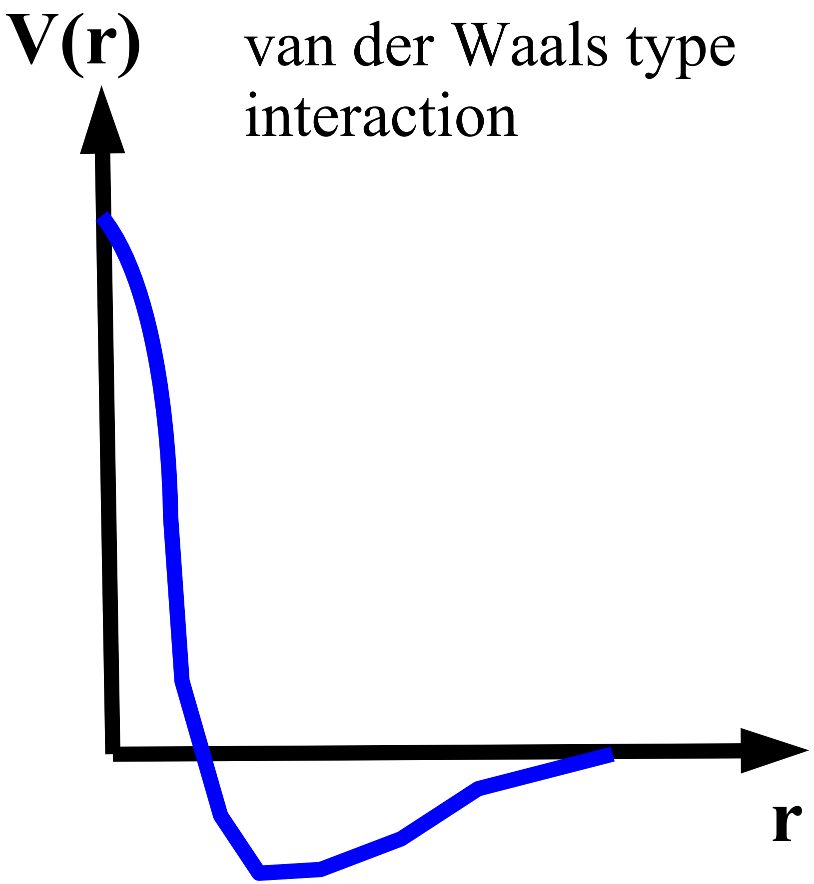
The QCD Phase Diagram
(from a theorist's perspective)

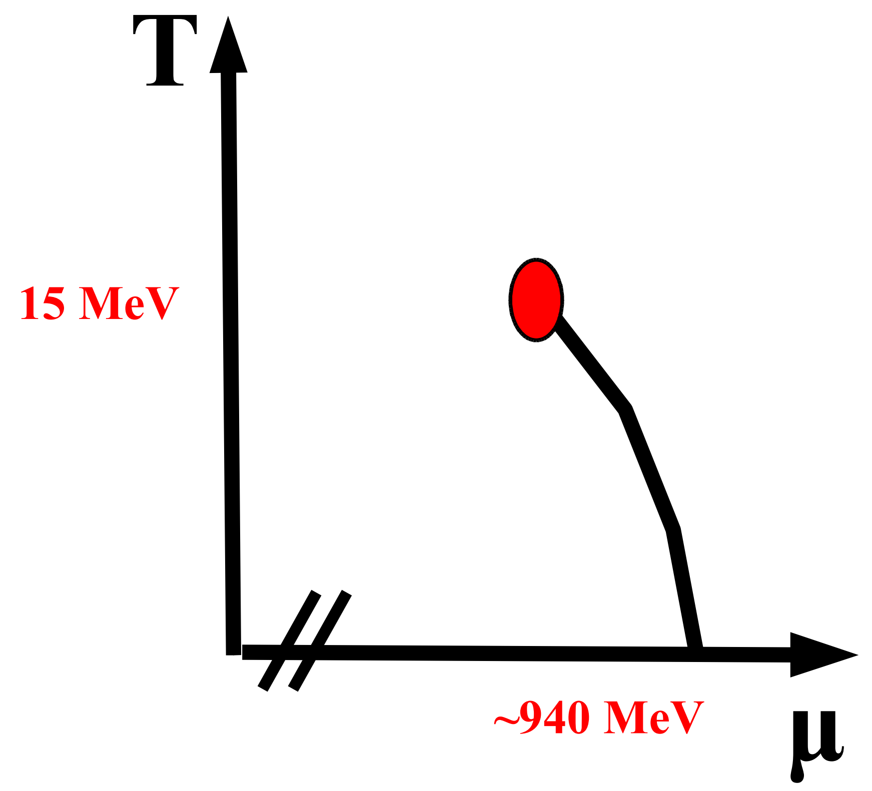


N.B.: Critical point of water: T_c=647.096 K, p_c = 22.064 MPa, ρ_c = 322 kg/m3

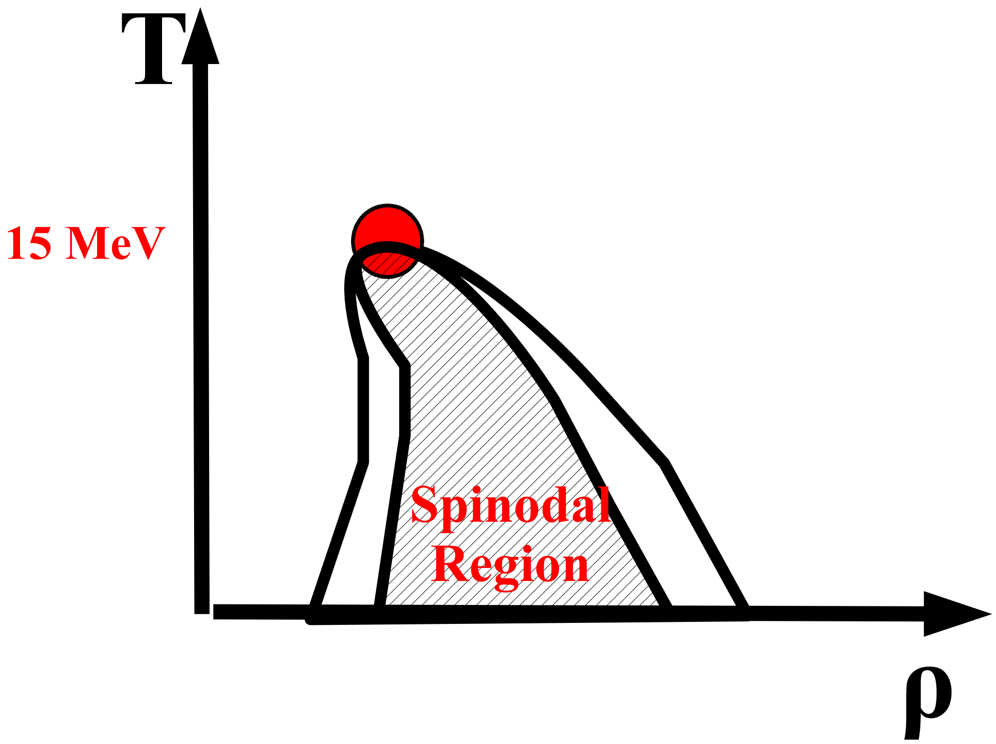
The Nuclear Liquid Gas Phase Transition

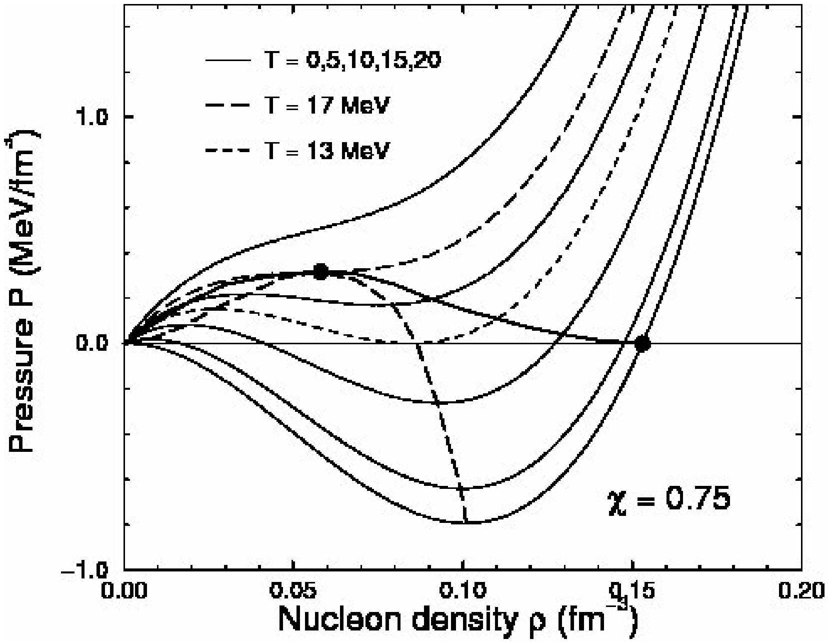
van der Waals type interaction





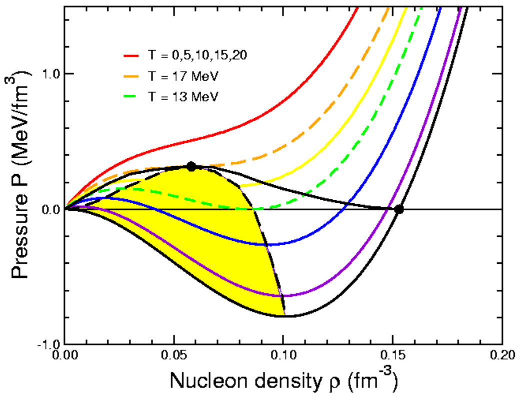
Nuclear Liquid-Gas Transition





Spinodal Multifragmentation

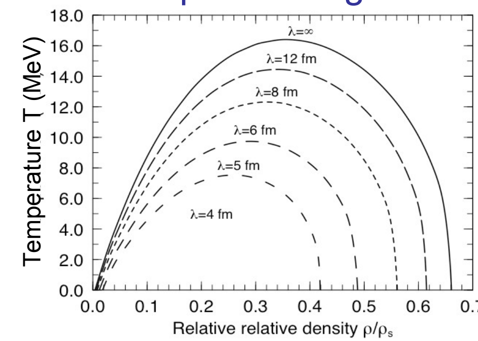
Nuclear EoS:

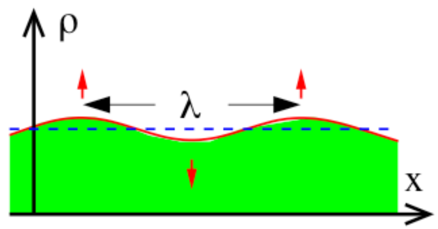


1st order phase transtion

Spinodal instability

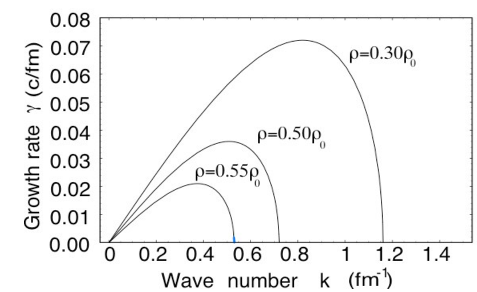
Spinodal region:



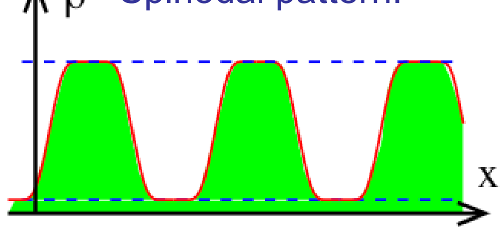


Density undulations may be amplified

Growth rates:



Spinodal pattern:



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263

Fragments
≈ *equal!*

Highly non-statistical => Good candidate signature

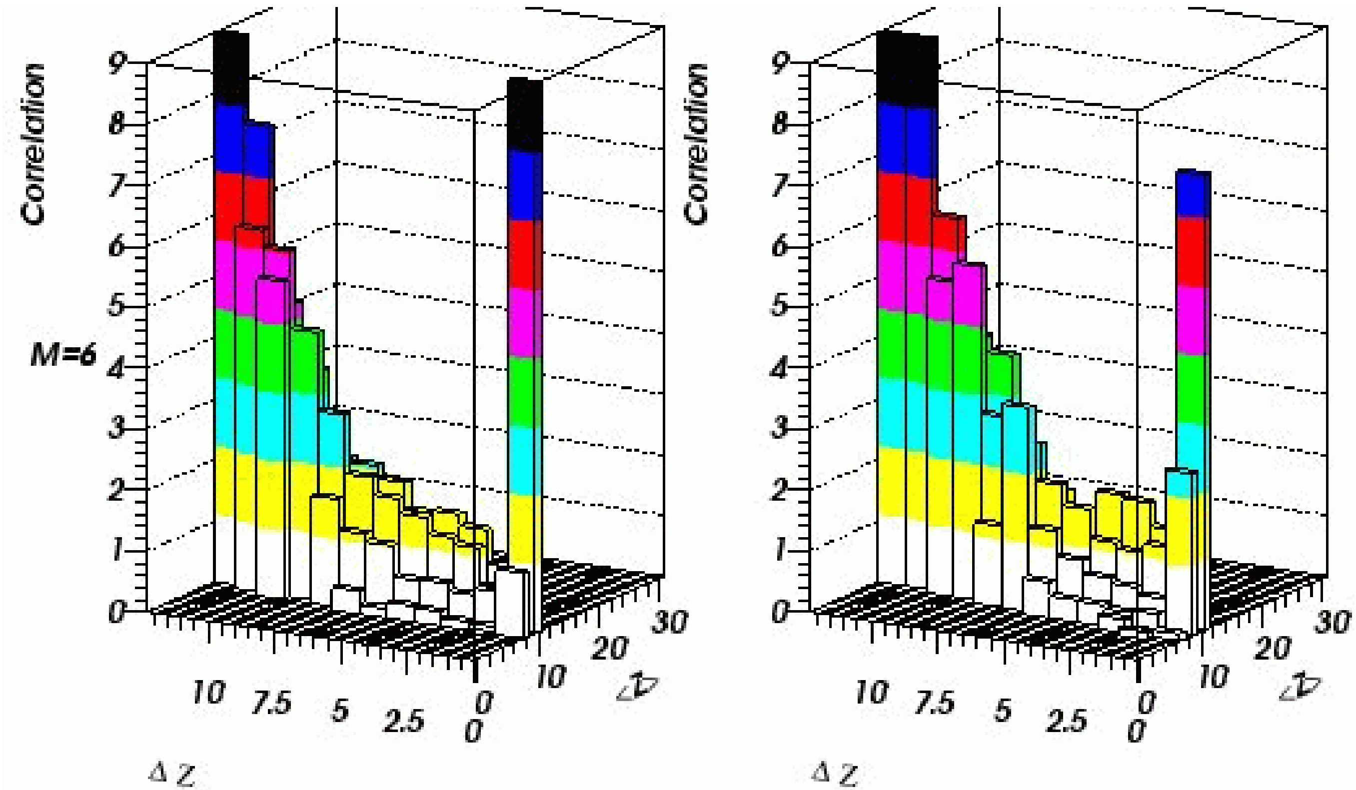
J. Randrup

Spinodal decomposition in nuclear multifragmentation

32 MeV/A Xe + Sn (b=0)

(select events with 6 IMFs)

Bin wrt $\begin{cases} \langle Z \rangle : & \text{average IMF charge} \\ \Delta Z : & \text{dispersion in IMF charge} \end{cases}$



Experiment (*INDRA @ GANIL*)

Borderie *et al*, PRL 86 (2001) 3252

Theory (*Boltzmann-Langevin*)

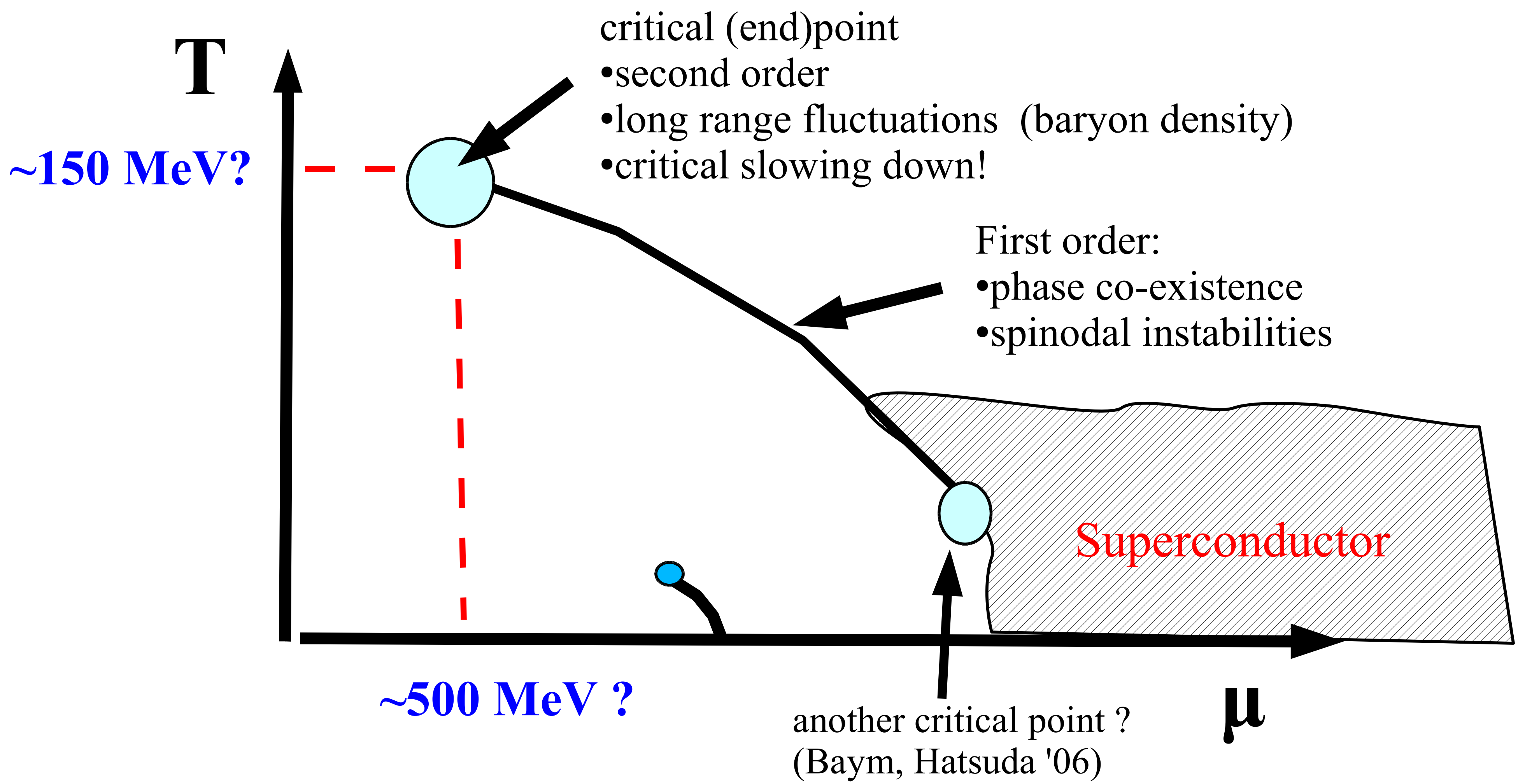
Chomaz, Colonna, Randrup, …

J. Randrup

Summary Nuclear Liquid Gas

- Conceptually very straightforward
  - Force of van der Waals type
- Signs for co-existence have been found
  - Spinodal
  - Systematics of fragment distribution follows Fisher model
    - Extrapolate to critical point ?
- Phases are rather well defined
- >20 years of work !

The QCD Phase Diagram
(from theory)

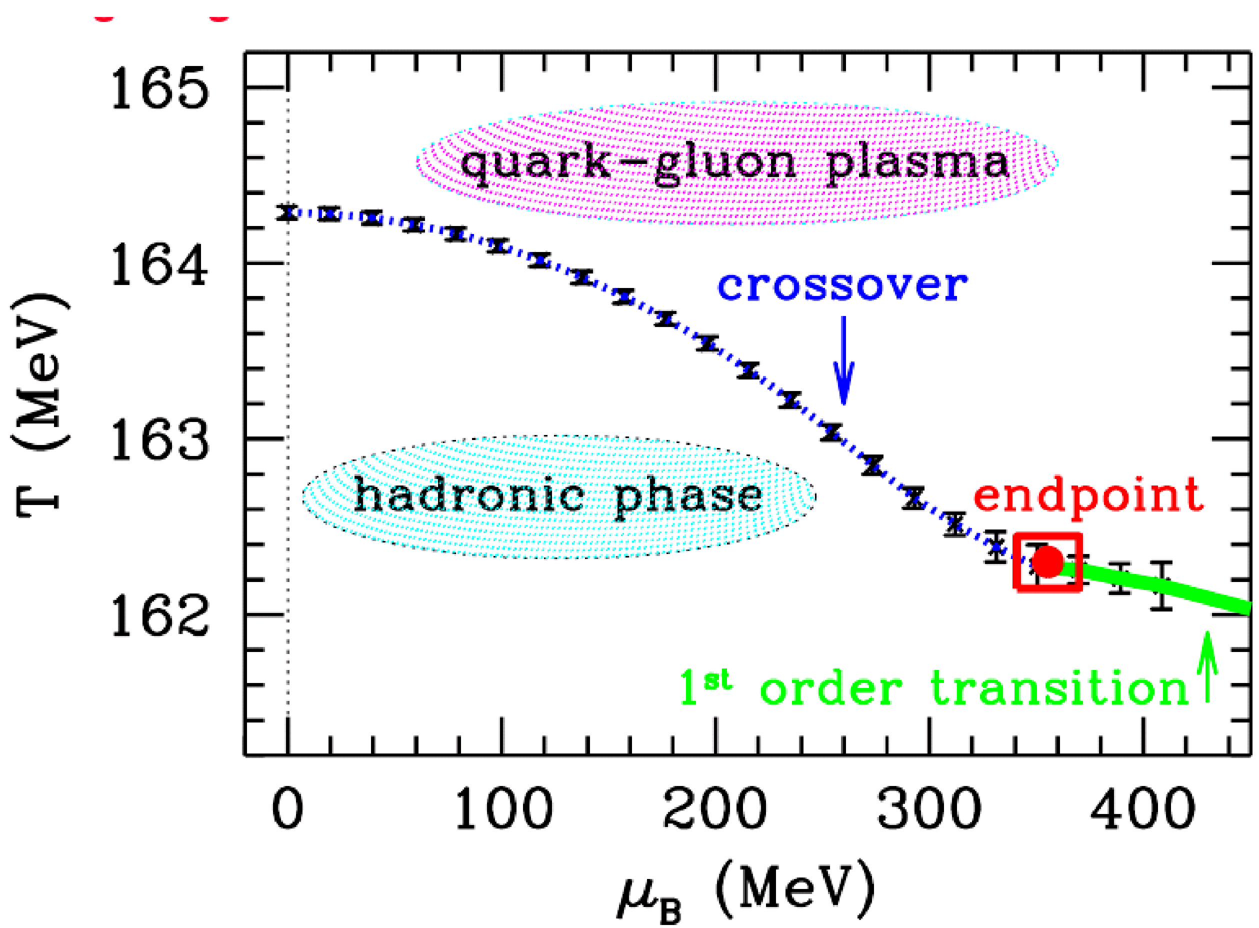


What do we know about QCD CP / Phase co-existence

- Lattice:
  - Reweighting: CP (+)
  - Taylor expansion: radius of convergence (+/-)
  - Curvature of critical line (-)
- Models:
  - Nambu/Sigma models (+) (high mu)
    - Even two critical points (-> Kapusta)
  - Vector coupling (+/-/0)! (-> Fukushima)

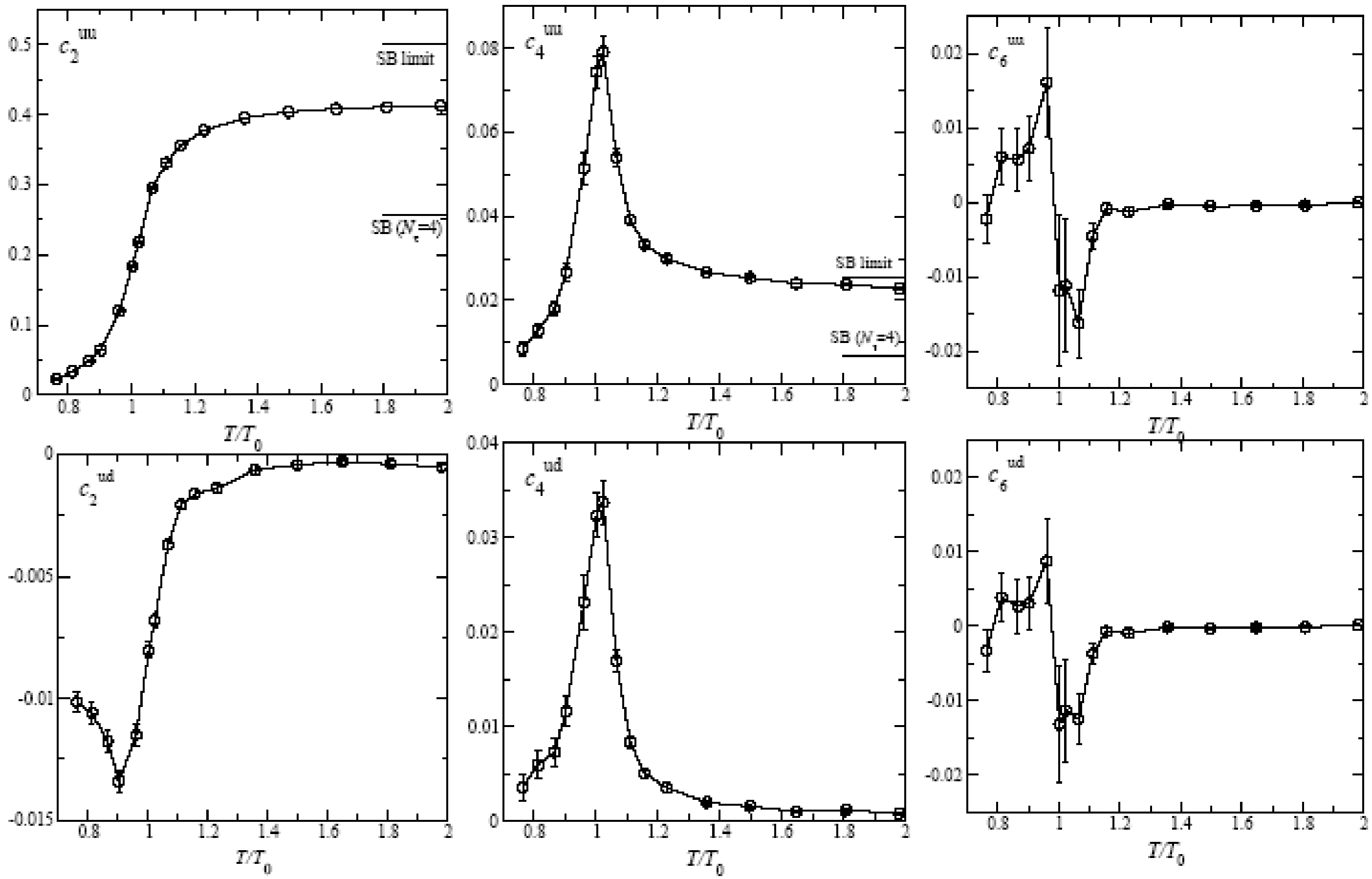
Re-Weighting

Fodor and Katz,JHEP 0404 (2004) 050



Lattice-QCD susceptibilities

$$\frac{\chi(T,\mu_q)}{T^2} = 2c_2 + 12\, c_4 \left(\frac{\mu_q}{T}\right)^2 + 30\, c_6 \left(\frac{\mu_q}{T}\right)^4 + \ldots$$



Rule of thumb:

$$c_n \sim \langle X^n \rangle$$
$$X = B, Q, S, \ldots$$

Alton et al, PRD 66 074507 (2002)

Hadronic fluctuations and the QCD critical point

- **Consequences for the phase diagram:**
  the radius of convergence

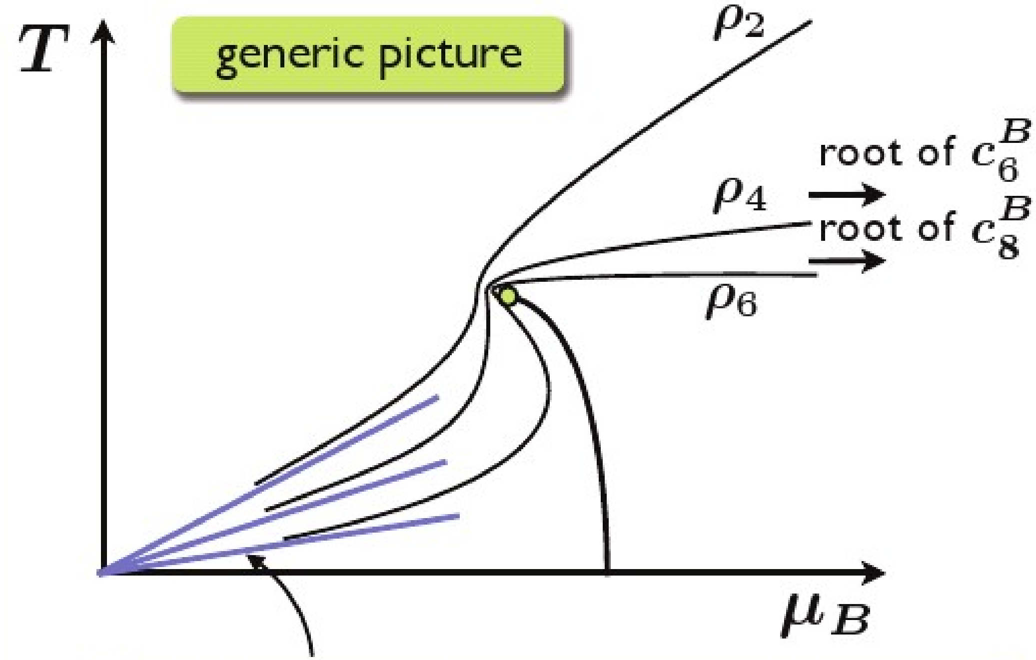
The radius of convergence can be estimated from the Taylor coefficients of the pressure:

$$\rho = \lim_{n\to\infty} \rho_n$$

with

$$\rho_n = \sqrt{\frac{c_n^B}{c_{n+2}^B}}$$

- for $T > T_c, \quad \rho_n \to \infty$
- for $T < T_c$, ρ_n is bound by the transition line



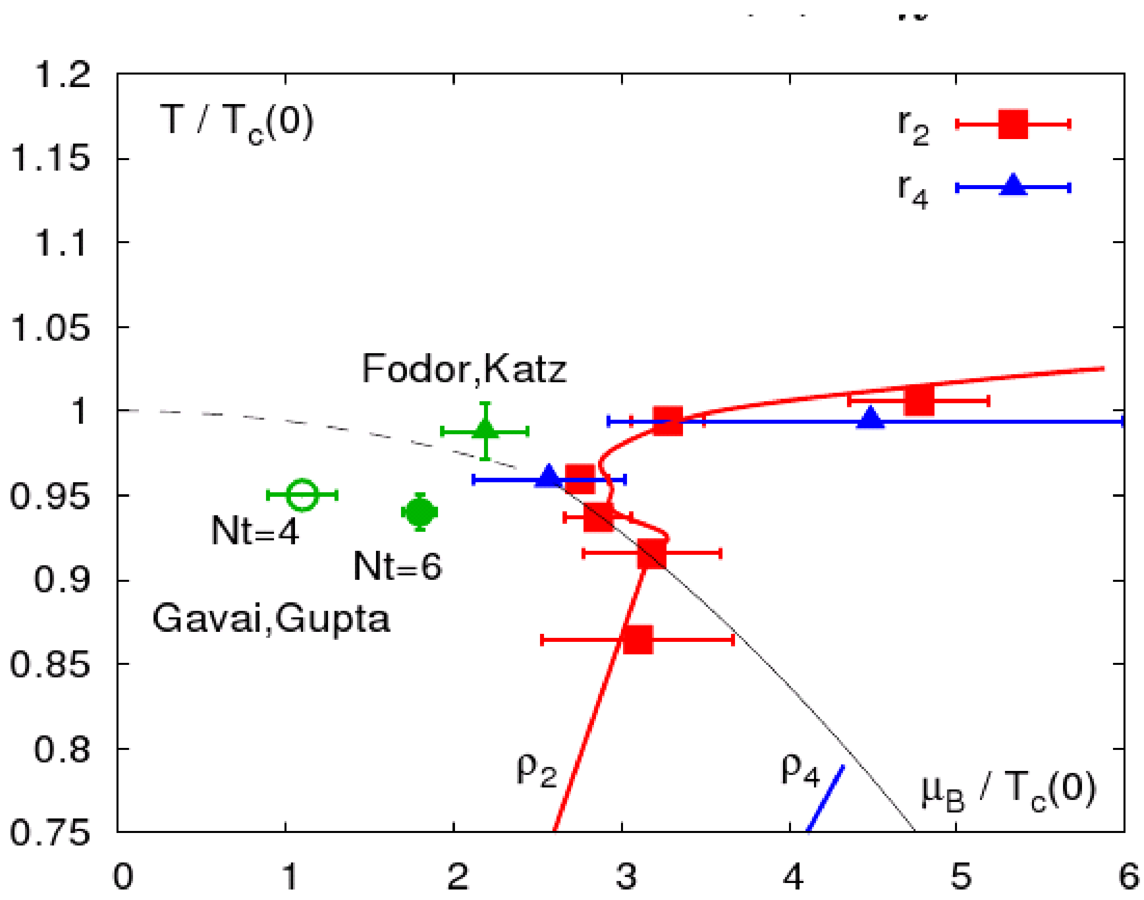
The Resonance gas limit:

$$\frac{p}{T^4} = G(T) + F(T)\cosh\left(\frac{\mu_B}{T}\right)$$

$$\longrightarrow \rho_n = \sqrt{1/(n+2)(n+1)}$$

$\longrightarrow$ look for non-monotonic behavior in the radius of convergence

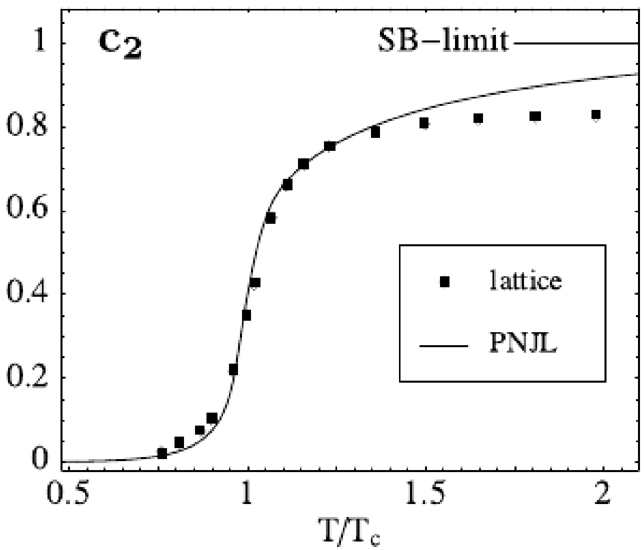
Radius of convergence

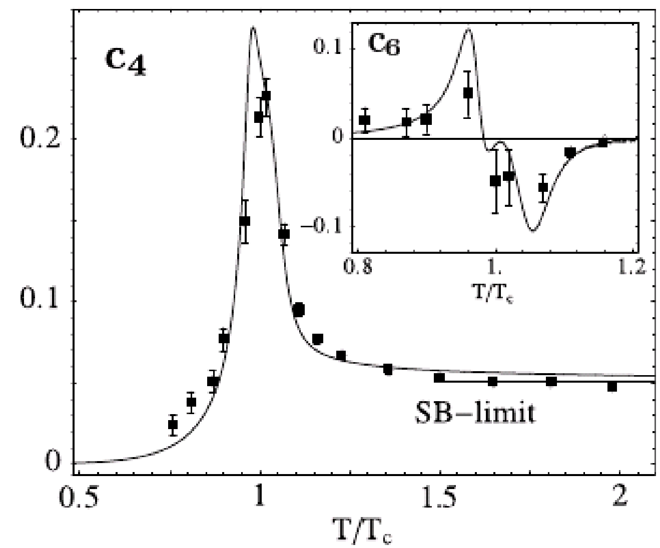


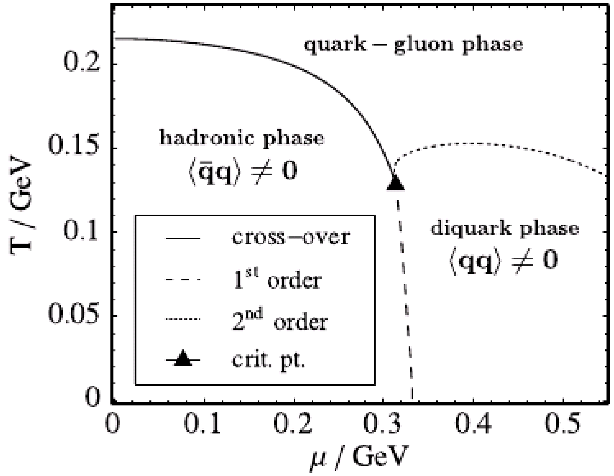
Note: n=2,4 and NOT infinity!

PNJL model

C. Ratti et al







Reproduces suceptibilities!

Critical point at $\mu_B \sim 900$ MeV!

Toy model be de Forcrand:
No CP but susceptibilities
according to Lattice

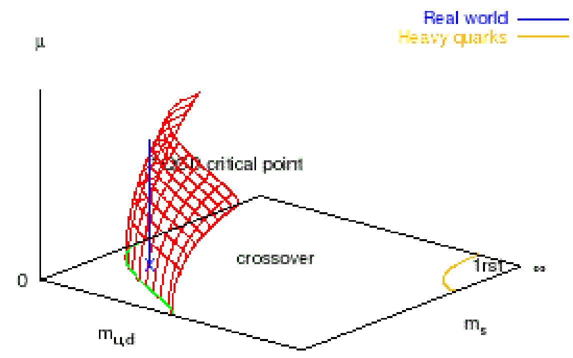
Lattice and the critical point

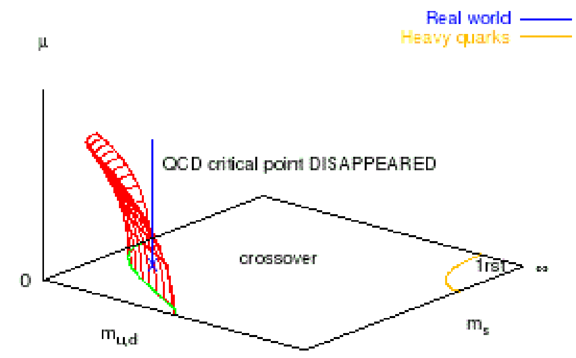
Forcrand, Philipsen

See talk by O. Philipsen

A non-standard scenario: no critical point?

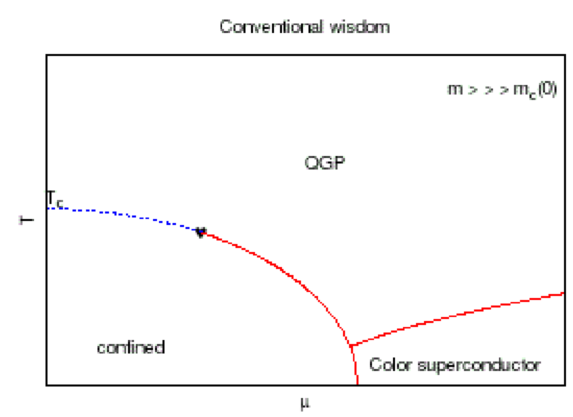
sign of $c_1 = \left.\frac{dm_c(\mu)}{d\mu^2}\right|_{\mu=0}$





If Phase transition is not chiral but ”liquid gas” and ”anchored at low T high μ the ”conventional scenario is still ok.

Question/Challenge:
Can we find a ROBUST phasetransition at high μ, e.g. such as in the nuclear liquid gas case?

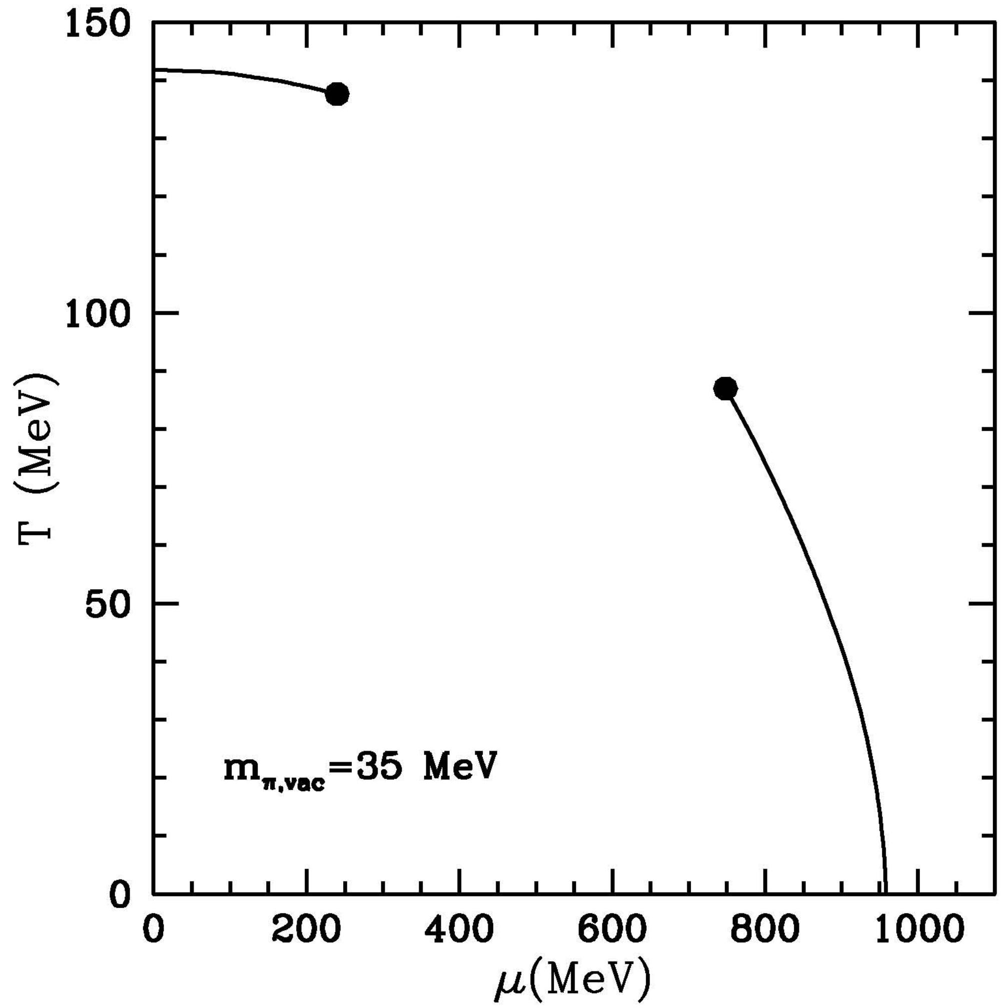


Similar effects due to vector coupling in PNJL (K. Fukushima)

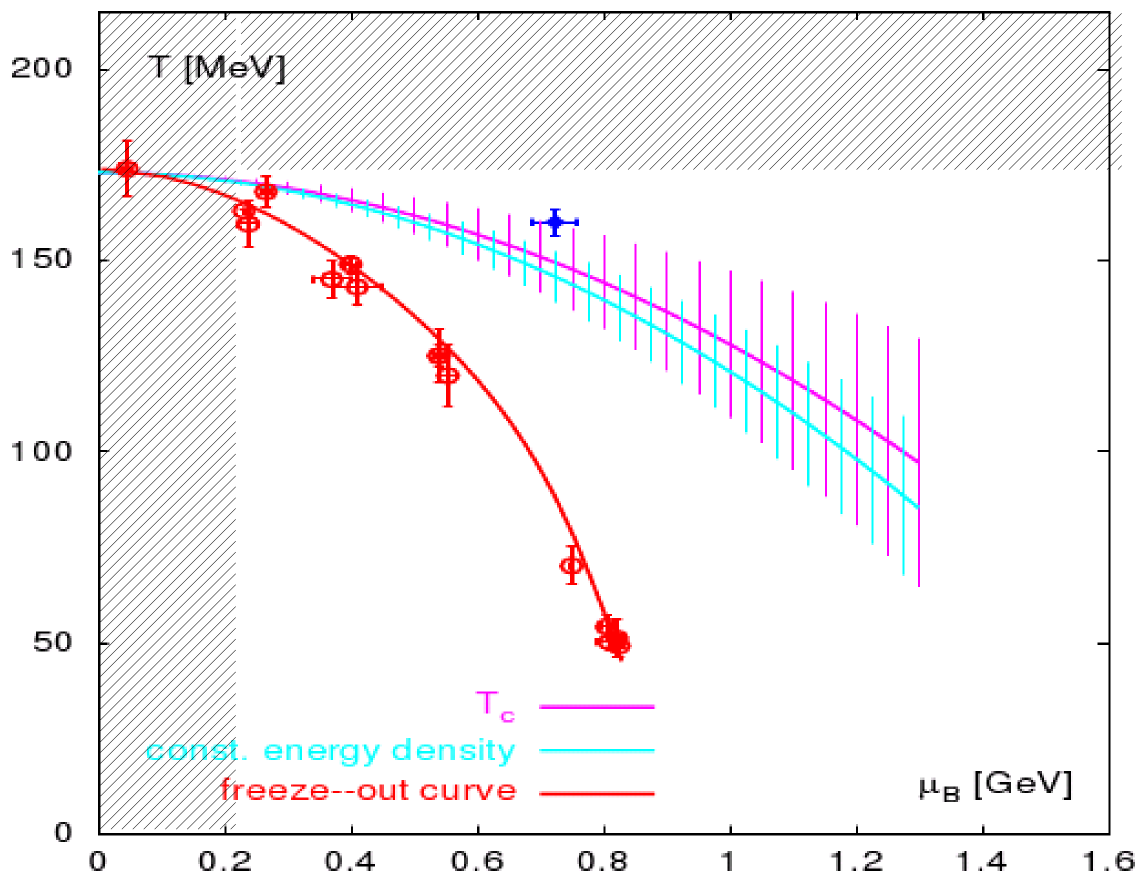
Sigma model with Quarks:

J. Kapusta, QM2009

Two critical points!



The QCD Phase Diagram
(“exclusion Plot”)



Towards data

Fluctuations and Correlations in thermal system

e.g. Lattice QCD

$$Z = Tr\left[\exp\left(-\beta\left(H - \mu_Q Q - \mu_B B - \mu_S S\right)\right)\right]$$

Mean : $\langle \alpha \rangle = T \frac{\partial}{\partial \mu_\alpha} \log(Z) = -\frac{\partial}{\partial \mu_\alpha} F$

$\alpha, \beta = Q, B, S$

Variance: $\langle (\delta \alpha)^2 \rangle = T^2 \frac{\partial^2}{\partial \mu_\alpha^2} \log(Z) = -T \frac{\partial^2}{\partial \mu_\alpha^2} F$

Co-Variance: $\langle (\delta \alpha)(\delta \beta) \rangle = T^2 \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} \log(Z) = -T \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} F$

Susceptibility: $\chi_{\alpha\beta} = -\frac{1}{V} \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} F = -\frac{1}{V} \frac{\partial}{\partial \mu_\alpha} \langle \beta \rangle$

Susceptibilities and Phasetransitions

$$Z = Tr[\exp(-\beta(H-\mu N))]$$

Susceptibility: $\chi \sim \frac{1}{V}\frac{\partial^2}{\partial \mu^2}\log(Z) = \frac{1}{V}(\langle N^2\rangle - \langle N\rangle^2)$

Poisson: $\chi \sim \frac{\langle N\rangle}{V}$ independent of volume ➡ $\langle(\delta N)^2\rangle = N \sim V$

In general: $\chi \sim \frac{1}{V}\int d^3x\, d^3y \langle\rho(x)\rho(y)\rangle_{connected} = \int d^3r \langle\rho(r)\rho(0)\rangle_{connected} \sim \xi^2$

$$\langle\rho(r)\rho(0)\rangle_{connected} \sim \frac{e^{(-r/\xi)}}{r} \qquad \xi = correlation\ length$$

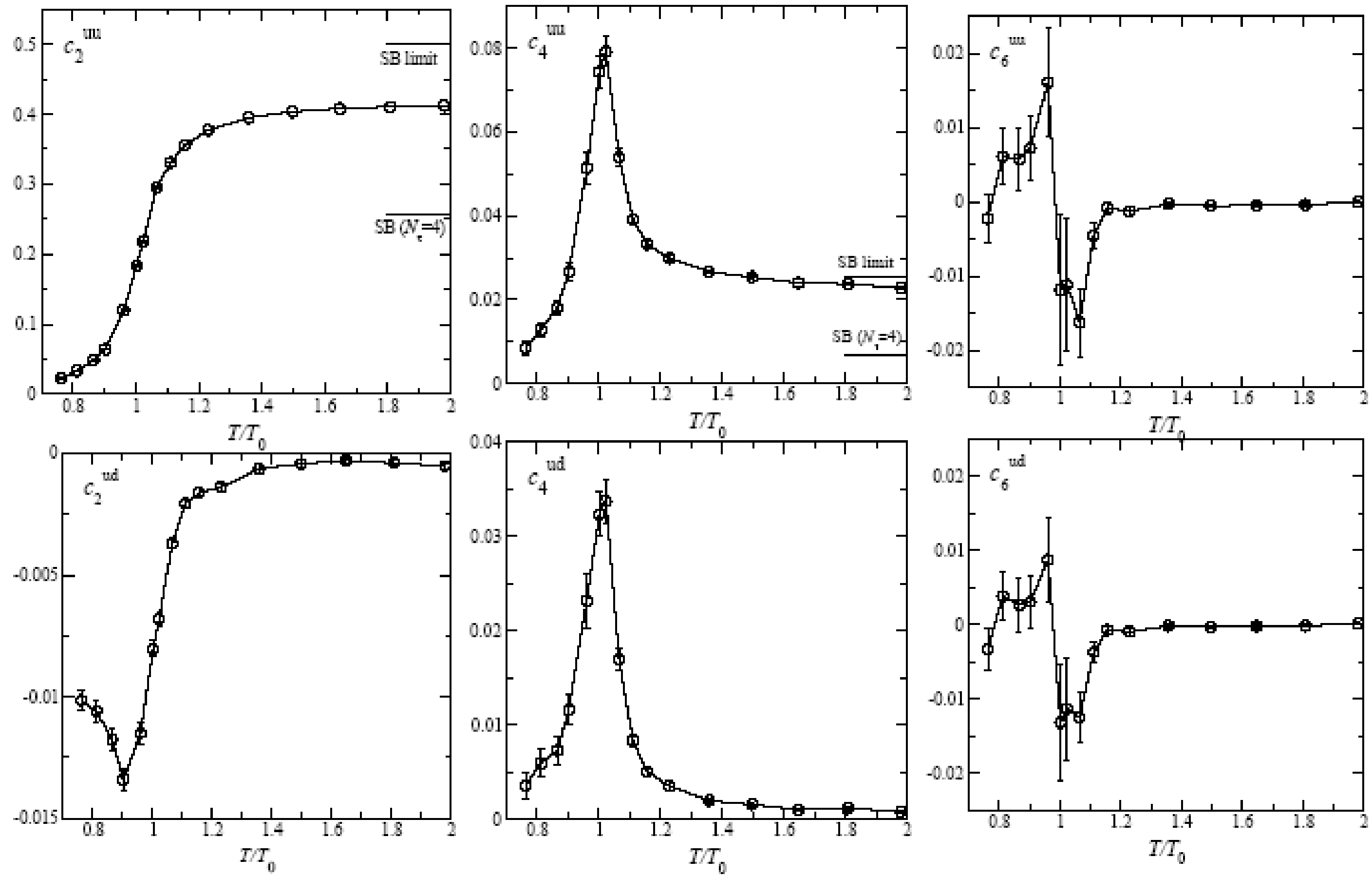
Cross-over: $\xi = const \rightarrow \chi = const \rightarrow \langle(\delta N)^2\rangle \sim V$

Second Order: $\xi \sim V^{(1/3)} \rightarrow \chi \sim V^{(2/3)} \rightarrow \langle(\delta N)^2\rangle \sim V^{(5/3)}$

First Order: $\langle\rho(r)\rho(0)\rangle = const \rightarrow \chi \sim V \rightarrow \langle(\delta N)^2\rangle \sim V^2$

Lattice-QCD susceptibilities

$$\frac{\chi(T,\mu_q)}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4 + \ldots$$



Rule of thumb:

$$c_n \sim \langle X^n \rangle$$
$$X = B, Q, S, \ldots$$

Alton et al, PRD 66 074507 (2002)

Susceptibilities and Observables

Susceptibility: $$\chi \sim \frac{1}{V}\frac{\partial^2}{\partial \mu^2}\log(Z) = \frac{1}{V}(\langle N^2\rangle - \langle N\rangle^2)$$

Fluctuations of some sort!

Cross-over: $$\xi = const \rightarrow \chi = const \rightarrow \langle(\delta N)^2\rangle \sim V$$

Second Order: $$\xi \sim V^{(1/3)} \rightarrow \chi \sim V^{(2/3)} \rightarrow \langle(\delta N)^2\rangle \sim V^{(5/3)}$$

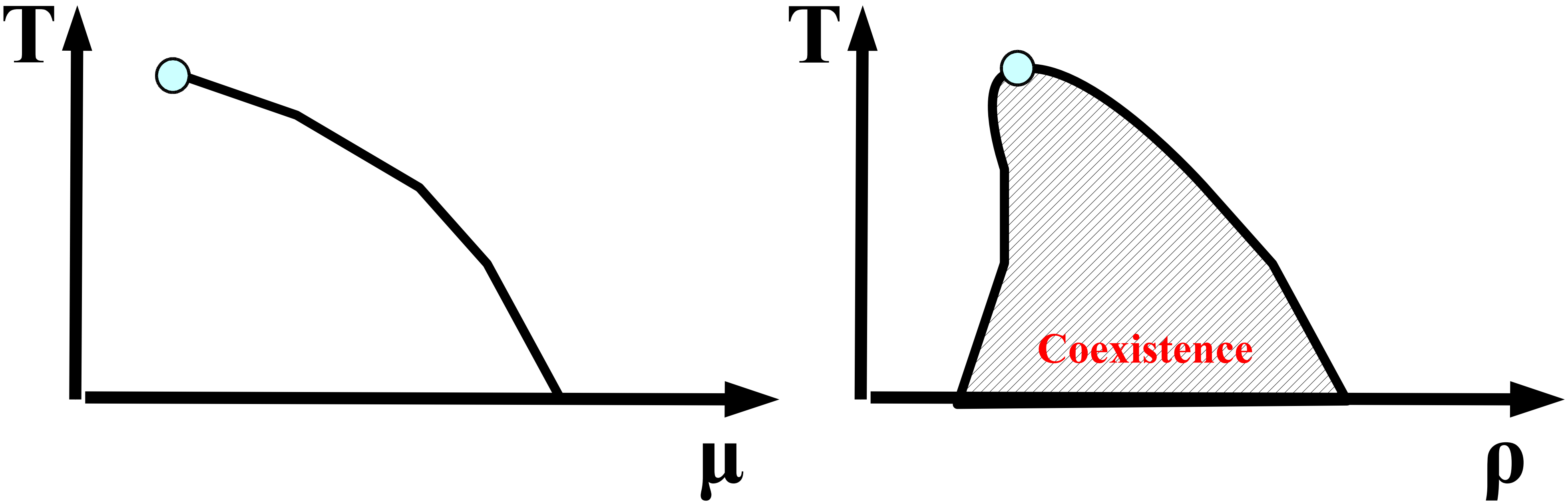
First Order: $$\langle \rho(r)\rho(0)\rangle = const \rightarrow \chi \sim V \rightarrow \langle(\delta N)^2\rangle \sim V^2$$

Since fluctuations diverge at phase transition any sort will do!

System size dependence?
NOT if correlation length < system size (critical slowing down)

Note: Co-variances also diverge! trigger?

Order Parameter

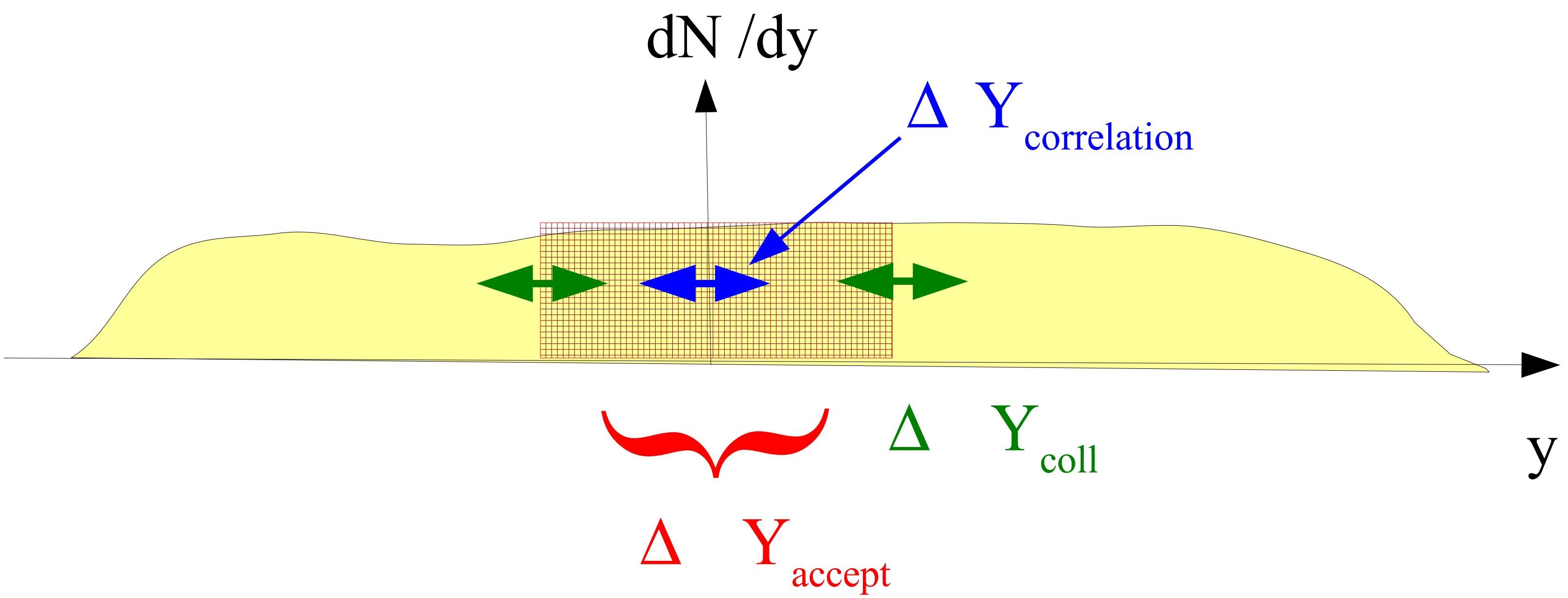


Baryon density is a good order parameter
density fluctuations are a good observable (theoretically...)

→

Baryon Number fluctuations also good in principle
global baryon number conservation is an issue at low energies

“Charge” fluctuations

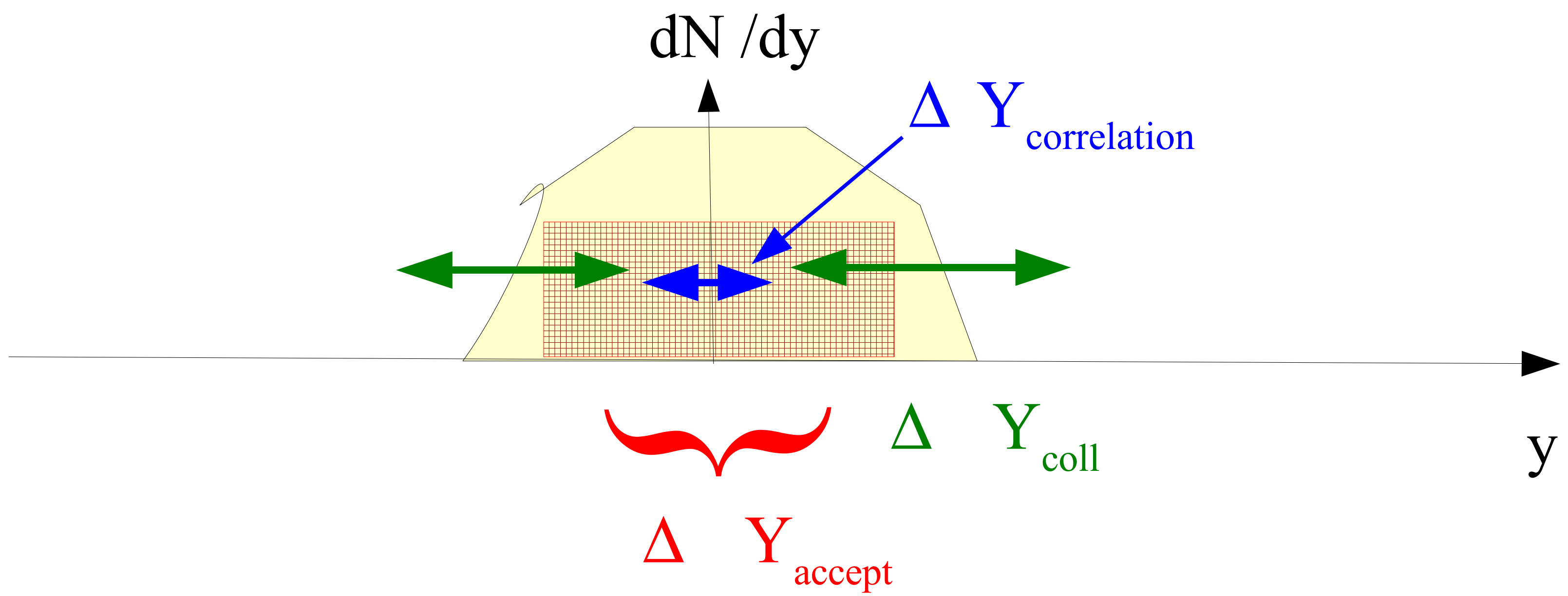


Condition for “charge” fluctuations:

1) $\Delta Y_{corrrelation} << \Delta Y_{accept}$ **(catch the physics)**

3) $\Delta Y_{total} >> \Delta Y_{accept} >> \Delta Y_{coll}$ **(keep the physics)**

"Charge" fluctuations at SPS and below



Condition for "charge" fluctuations:

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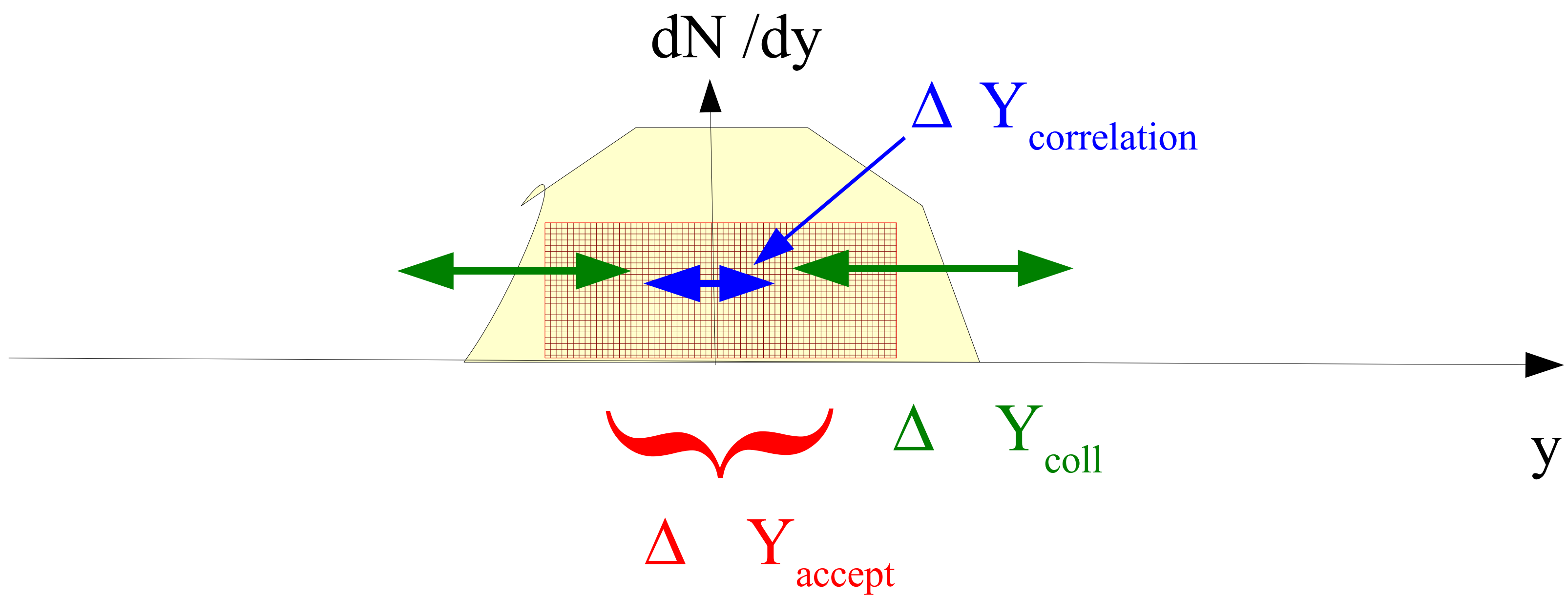
“Charge” fluctuations at SPS and below



~~Condition for “charge” fluctuations:~~

~~1)$\Delta \quad Y_{\text{corrrelation}} \ll \Delta \quad Y_{\text{accept}}$~~ **(catch the physics)**

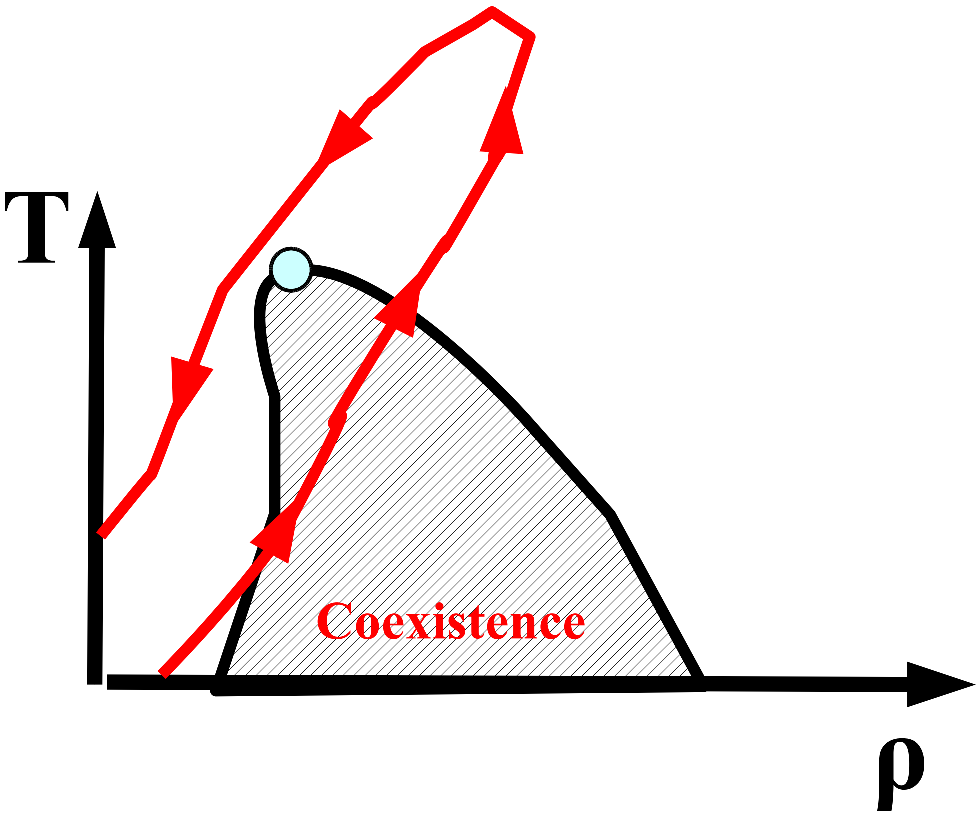
~~3)$\Delta \quad Y_{\text{total}} \gg \Delta \quad Y_{\text{accept}} \gg \Delta \quad Y_{\text{coll}}$~~ **(keep the physics)**

Baryon number conservation

- Affects all susceptibilities: Variance, Kurtosis
- Proton Fluctuations are also affected
  - Distinguish from Isospin fluctuations

- Still LARGE baryon DENSITY fluctuations

Critical point vs co-existence

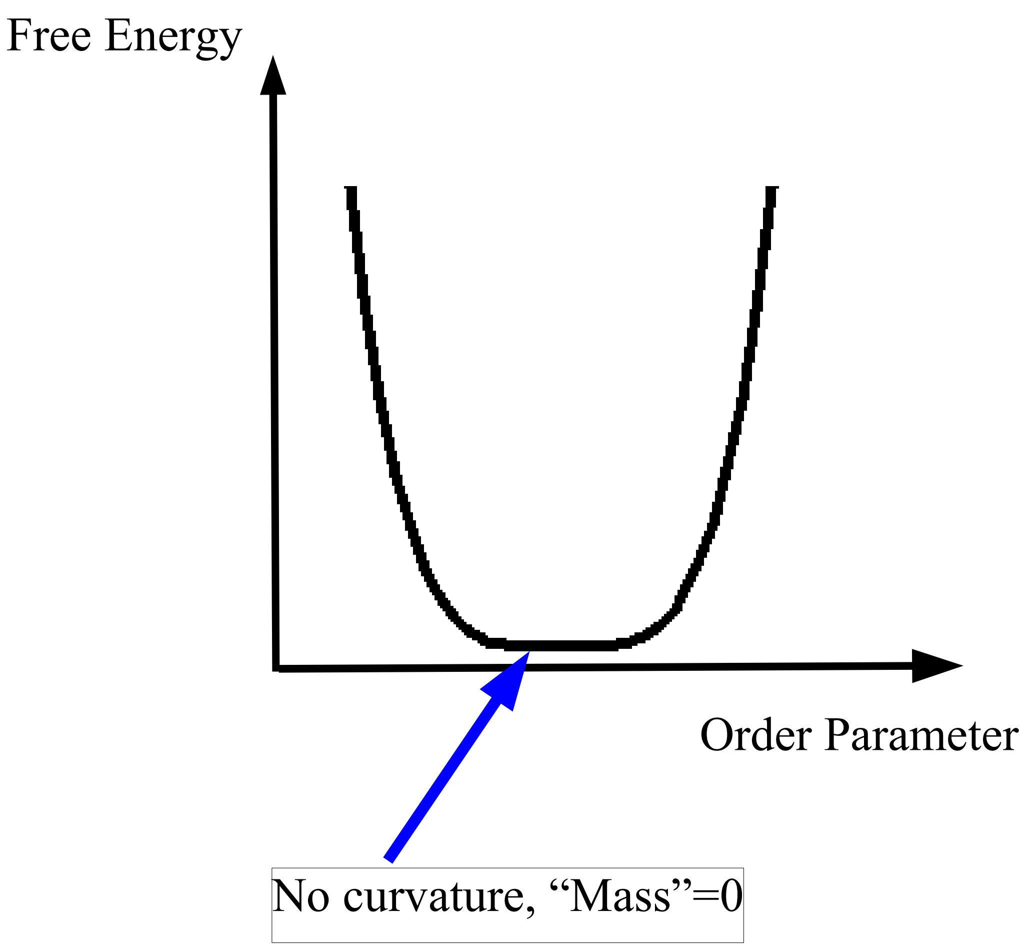


•Difficult to “hit” a point!

•Lesson learned from nuclear Liquid gas:

- Establish co-existence and extrapolate to CP
- Carefully chose energy such that system stalls in co-existence region

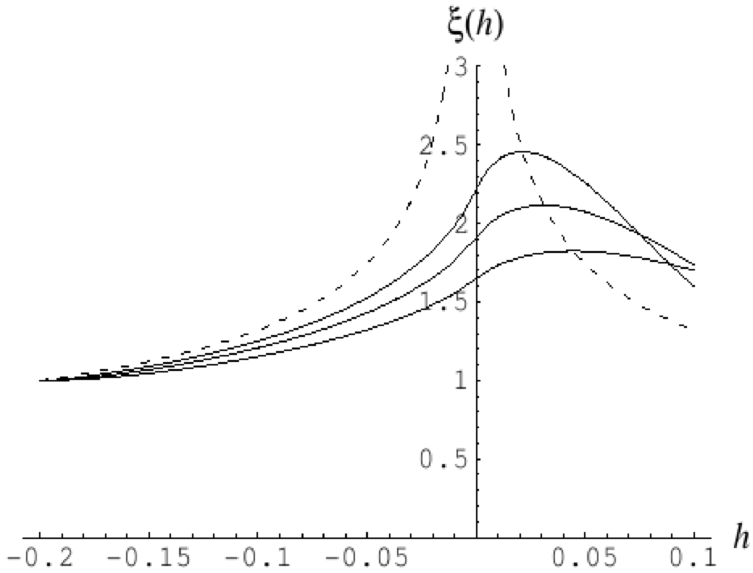
Critical Point = Second order



- Fluctuation of order parameter at all scales
- Diverging susceptibilities
  ~1/("Mass")^2
- Diverging correlation length
  ~1/("Mass")
- Universality
- Critical slowing down !

Critical Point = Second order

correlation length ~1/m_σ



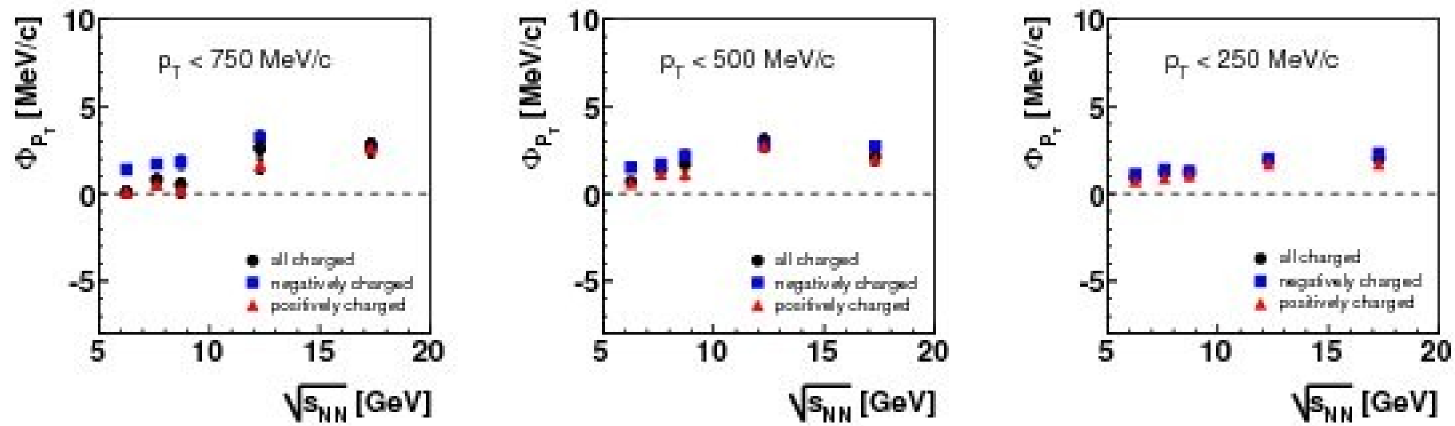
T/MeV 180 140 120

Bernikov, Rajagopal, hep-ph/9912274

- Critical slowing down
- limited sensitivity on model parameters
- Max. correlation length 2-3 fm
- Translates in **3-5%** effect in p_t-fluctuations

Expect:
Maximum in excitation function of p_t-fluctuations at low p_t

What does experiment say?



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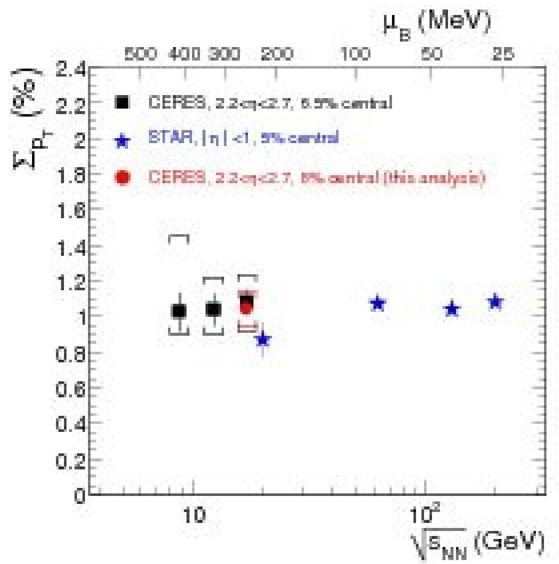
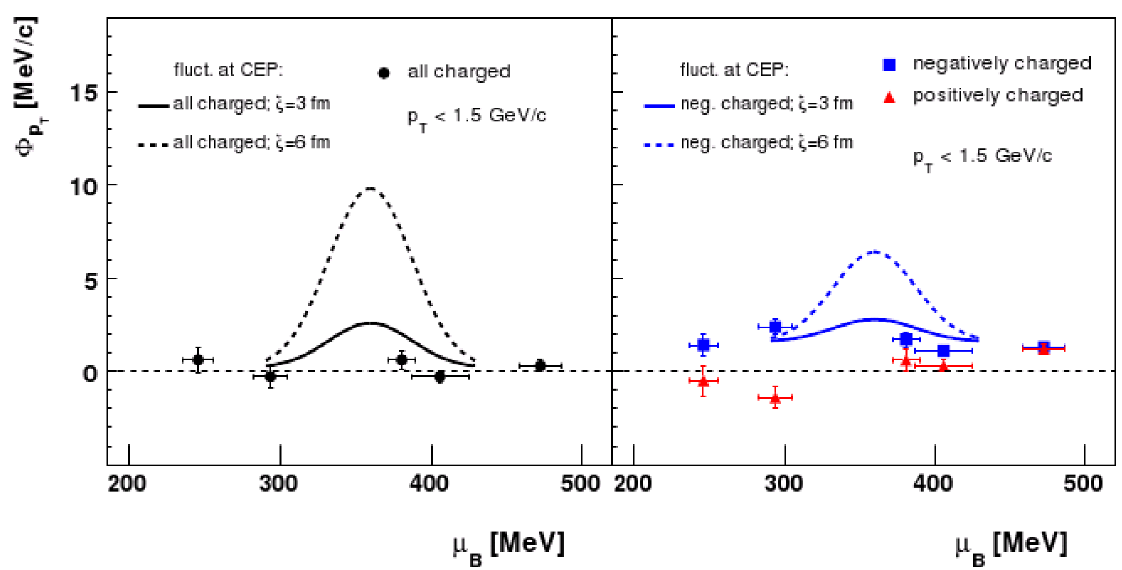




CERES & STAR

Higher cumulants?

Stephanov
arXiv:0809.3450

$$\omega_2 = \frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} \sim \xi^2$$

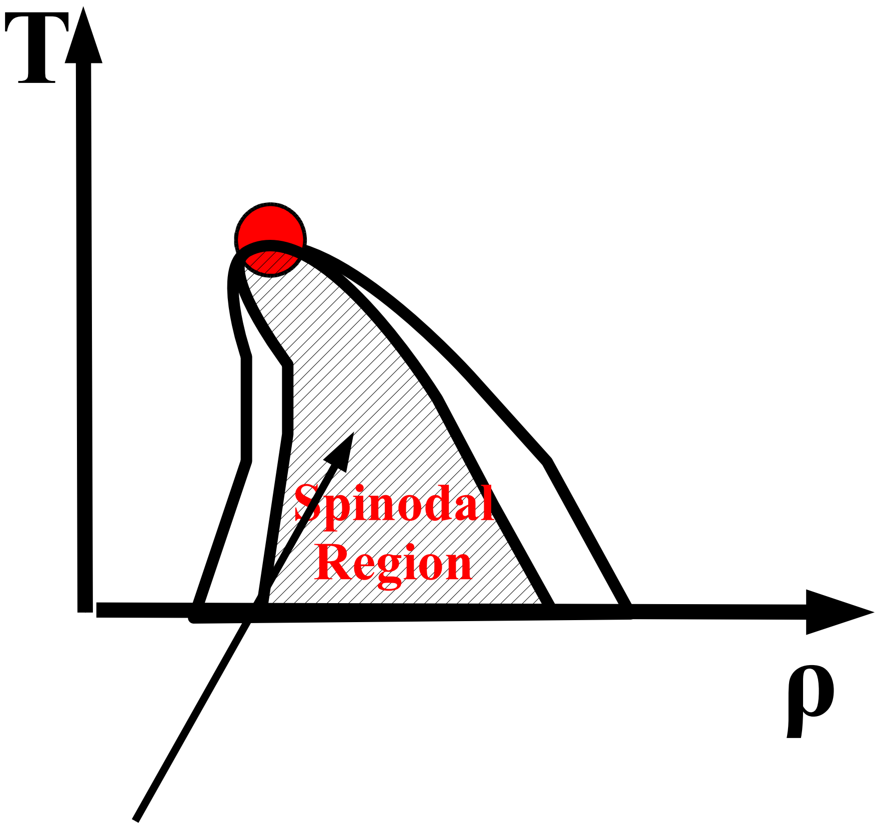
$$\omega_4 = \frac{\langle (\delta N)^4 \rangle}{\langle N \rangle} \sim \xi^7$$

Higher cumulants diverge with higher power:

5% in second order translates
20% in fourth order

Question: How does critical slowing down affect higher cumulants?

Co-existence region



System should spent long time
in spinodal region

See talk by J. Randrup

Growth rates γ_k

Small disturbance: $\varepsilon(x,t) = \varepsilon_0 + \delta\varepsilon(x,t)\ ,\ \delta\varepsilon \ll \varepsilon_0$

Evolution: $\partial_t^2 \delta\varepsilon(x,t) = \frac{\partial p_0}{\partial \varepsilon_0} \partial_x^2 \delta\varepsilon(x,t)$ => $\delta\varepsilon_k(x,t) \sim \mathrm{e}^{ikx - i\omega_k t}$

Dispersion relation: $\omega_k^2 = \frac{\partial p_0}{\partial \varepsilon_0} k^2 = -\gamma_k^2 k^2$ => $\gamma_k = |v_s|\, k$

Local average: $p(\boldsymbol{r}) = \langle p(\varepsilon(\boldsymbol{r}))\rangle$ $\omega_k^2 = \frac{\partial p_0}{\partial \varepsilon_0} g_k k^2\ ,\ g_k = \mathrm{e}^{-a^2 k^2/2}$

γ ~ k OK for small *k*
But what about *k* → ∞ ?
Ideal hydro has no scale!

a: smearing range suppresses large *k*

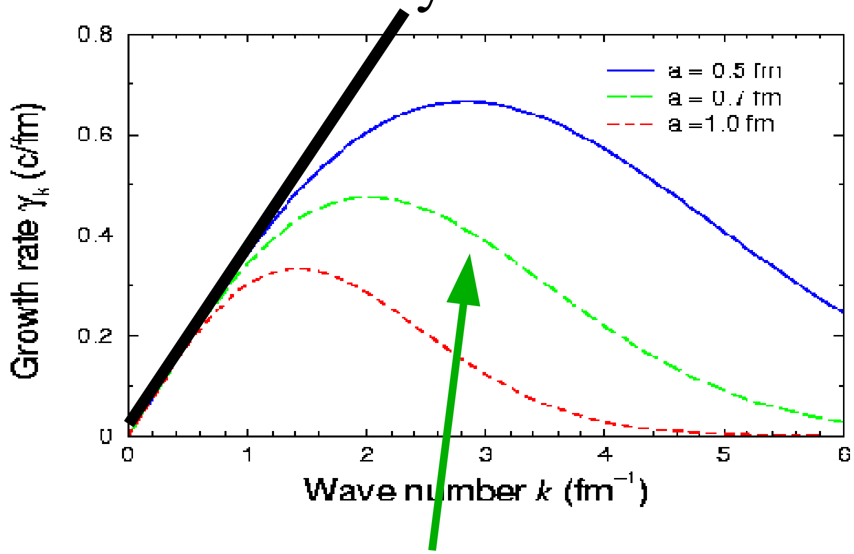
⇓

γ_k has a maximum

⇓

Spinodal pattern may develop

Ideal hydro



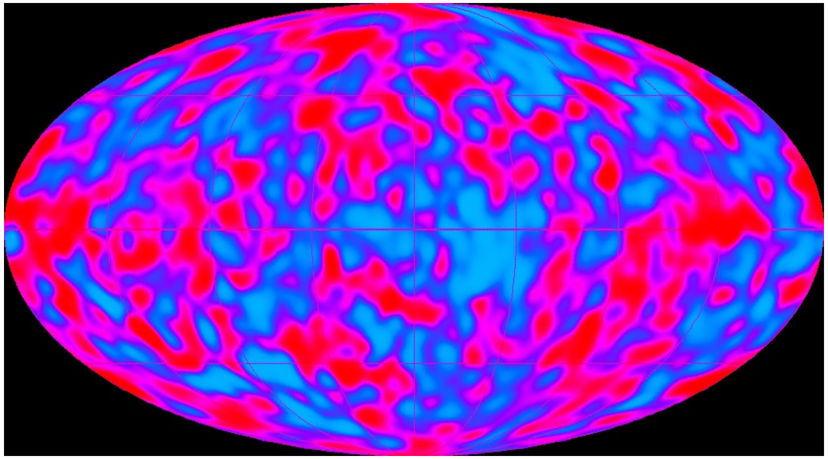
- *if* there is enough time!

Need a length scale!!
Interface tension from lattice?!

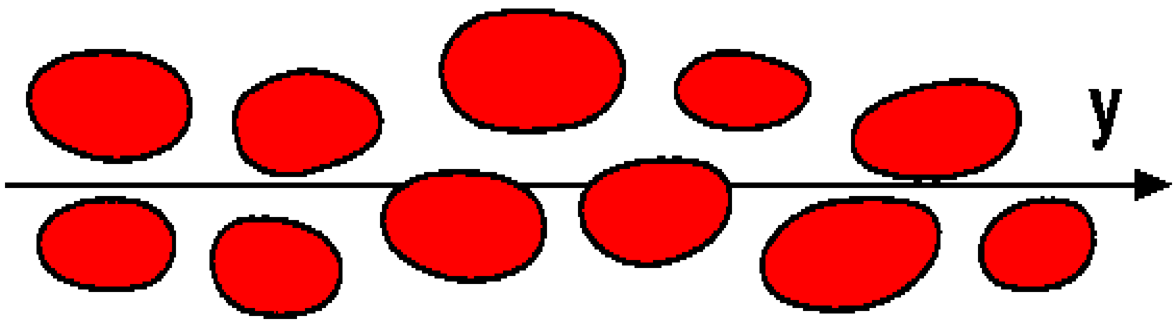
Talk byJ. Randrup

How to detect clumping?

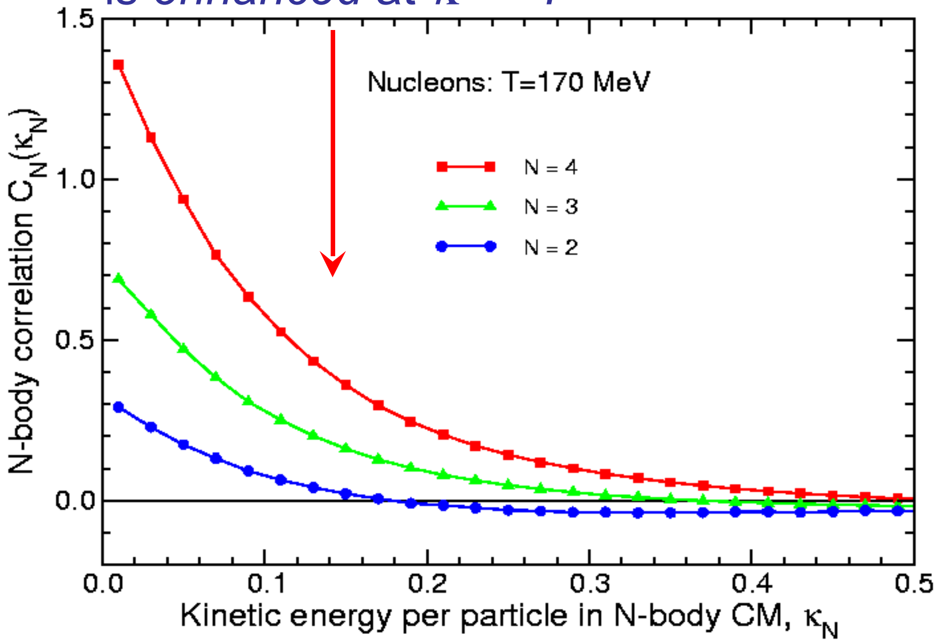
- No obvious candidates for clumps contrary to nuclear liquid gas
  - Kinematic correlations
  - Flavor correlations
- Fluctuations due to clumping
- Note: Hadrons are the dilute phase

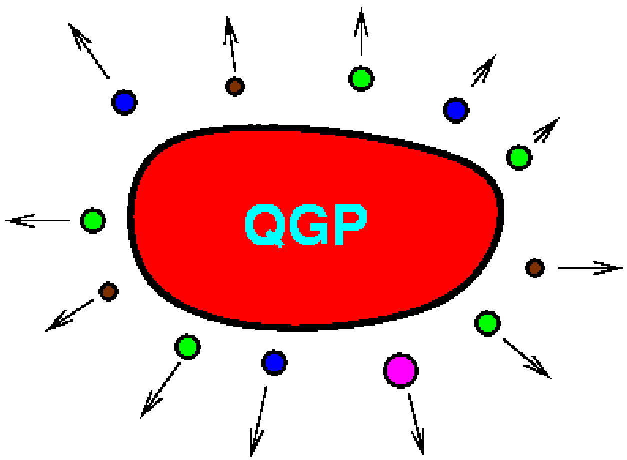


N-particle correlations



- is *enhanced* at $\kappa \approx T$

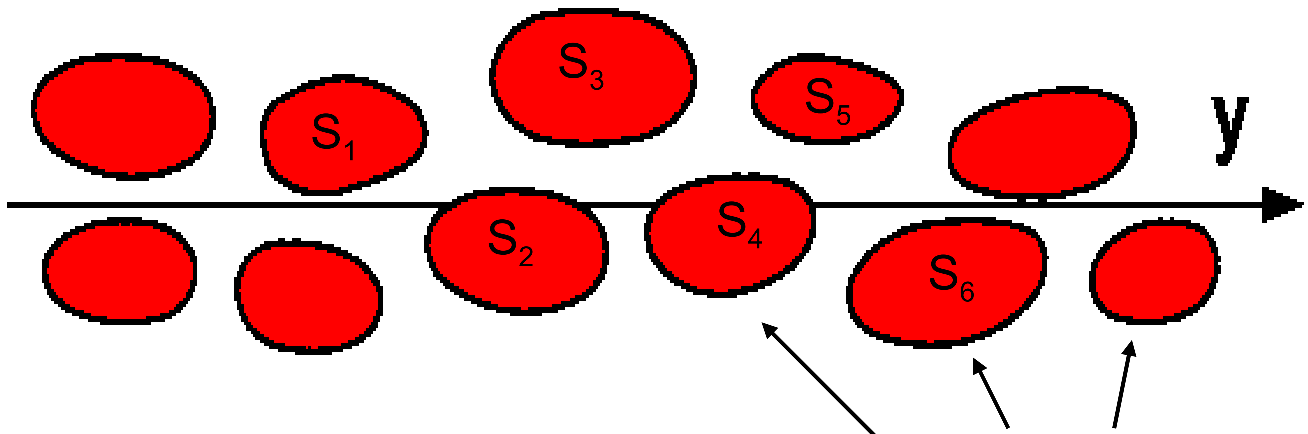




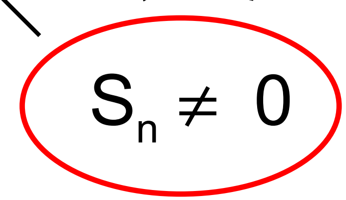
[J. Randrup, J. Heavy Ion Physics 22 (2005) 69]

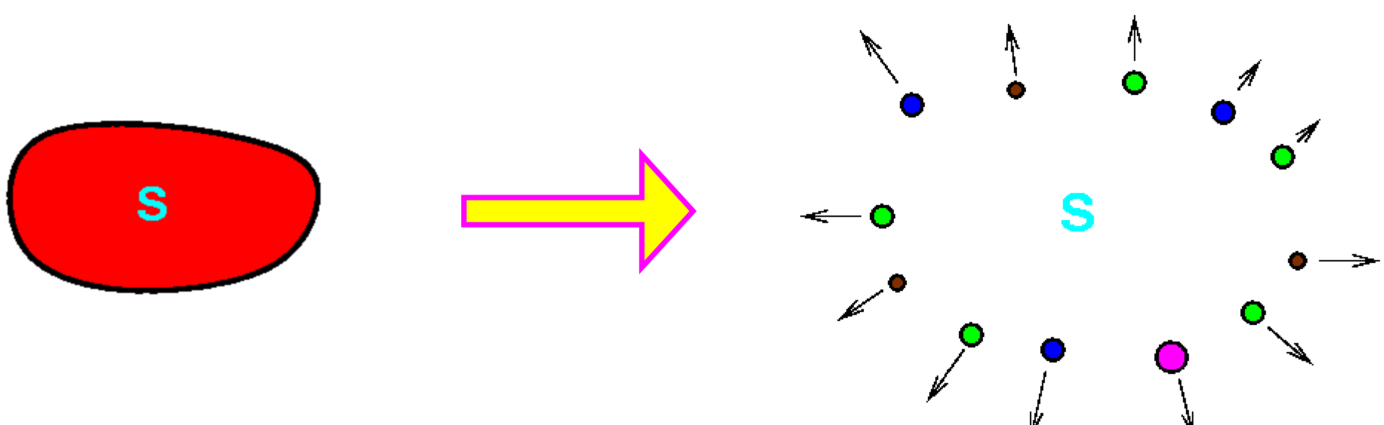
Strangeness correlations

The expanding system decomposes into plasma blobs
which each contain a certain amount of strangeness:



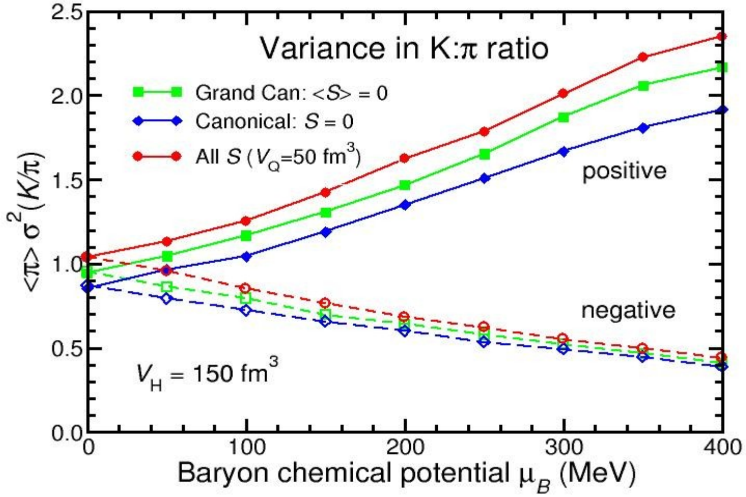
The hadronization of
each isolated blob
conserves strangeness:

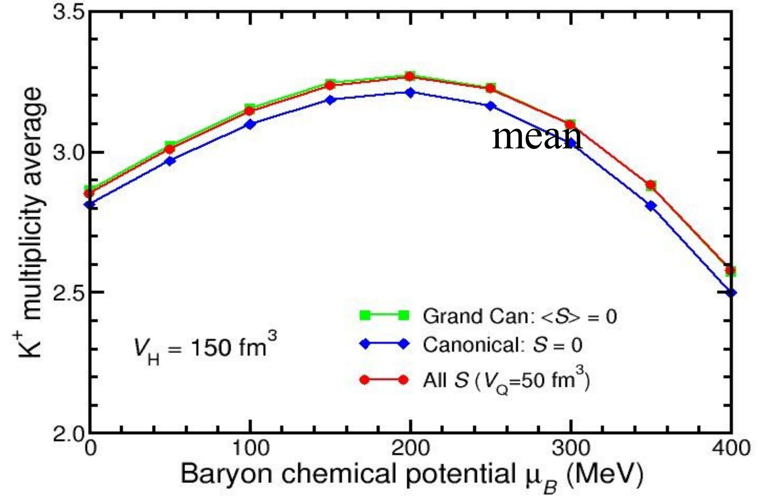




[V. Koch, A. Majumder, J. Randrup, Phys. Rev. C 72:064903,2005]

Some numbers





Variance: enhanced by ~10 %

$V_{QGP} = 50\ \mathrm{fm}^3$

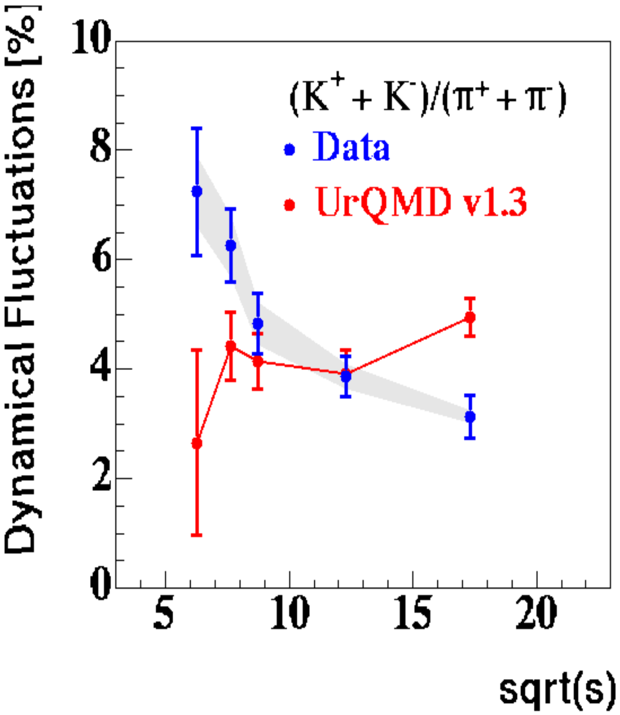
$V_{hadron} = 150\ \mathrm{fm}^3$

$T = 170$ MeV

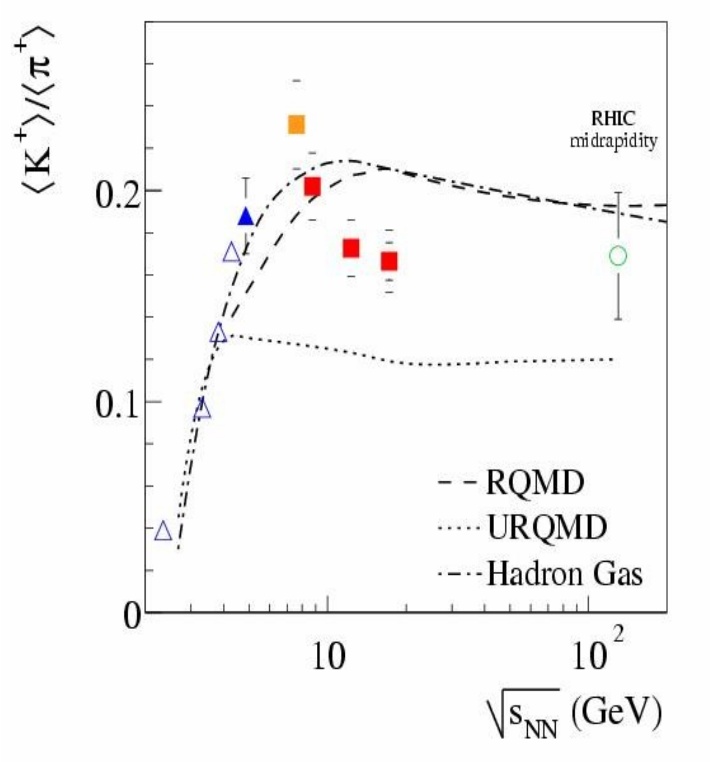
Generally: variance is more enhanced than mean

Strange things...

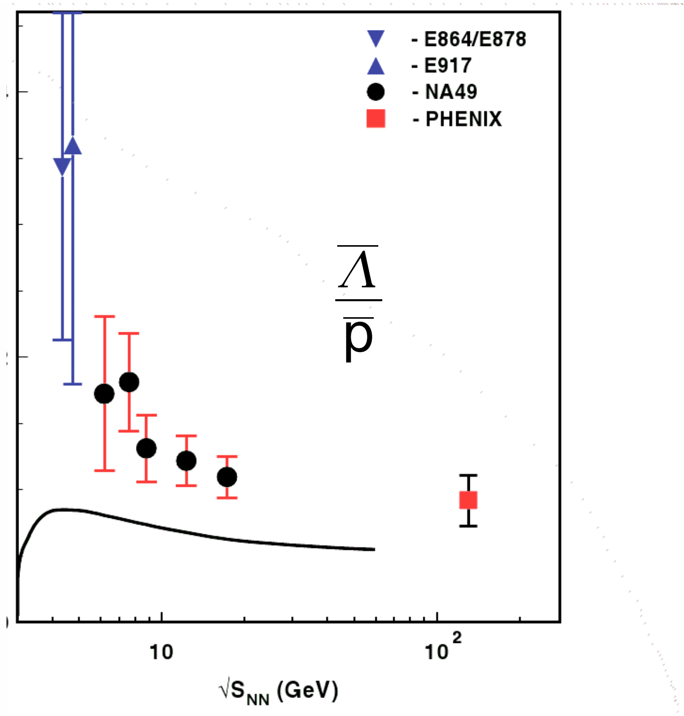
C. Roland, QM04, NA49



NA49



NA49, PRC73, 044910 (2006)

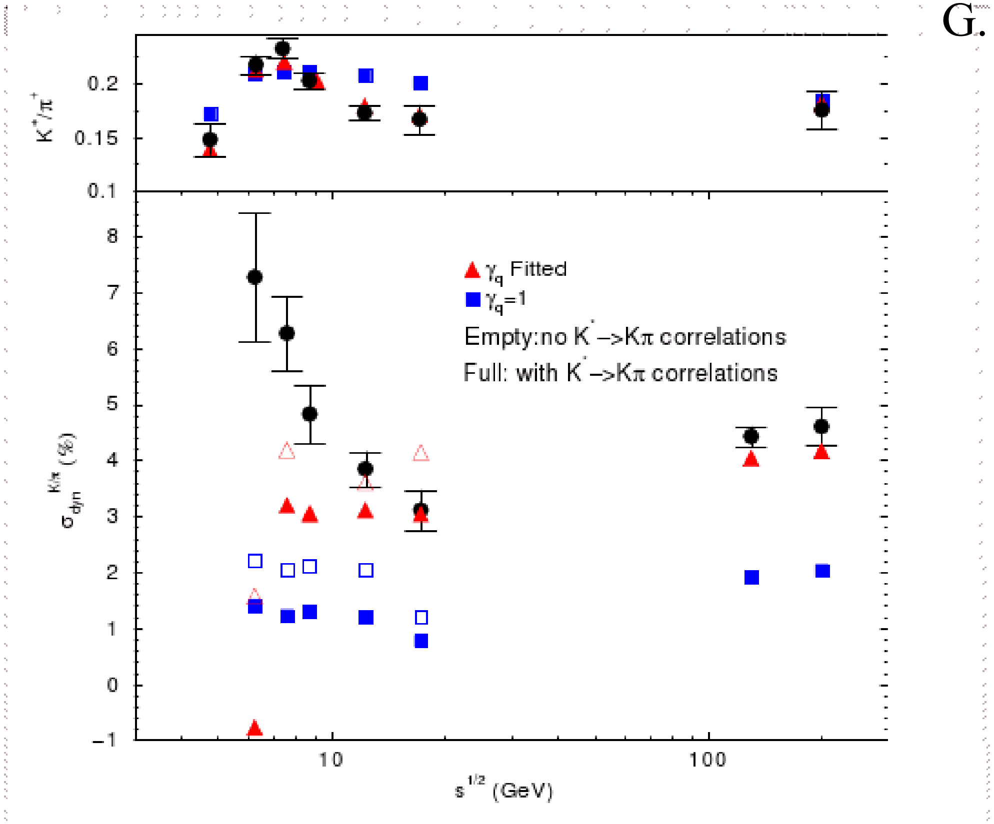


Hadron gas predictions

G. Torrieri, QM2006

$\langle \frac{K}{\pi} \rangle$

$\langle \delta (\frac{K}{\pi})^2 \rangle$



Some trivial effects...

w. T.Schuster and G. Westfall

$$\sigma^2_{dyn} = \frac{\langle \delta K^2 \rangle - \langle K \rangle}{\langle K \rangle^2} + \frac{\langle \delta \pi^2 \rangle - \langle \pi \rangle}{\langle \pi \rangle^2} - 2\frac{\langle \delta K \delta \pi \rangle}{\langle K \rangle \langle \pi \rangle}$$

$$= \frac{w_{KK} - 1}{\langle K \rangle} + \frac{w_{\pi\pi} - 1}{\langle \pi \rangle} - 2\frac{w_{K\pi}}{\sqrt{\langle K \rangle \langle \pi \rangle}}$$

$$\sim 1 \, / \, (\underline{\text{accepted}} \text{ Multiplicity})$$

$$w_{AB} \equiv \frac{\langle \delta A \delta B \rangle}{\sqrt{\langle A \rangle \langle B \rangle}}$$

Scaled correlation independent of multiplicity

Scaling prescriptions

Poisson scaling:
$$\sigma_{\rm dyn}\left(\sqrt{s}\right) = \sigma_{\rm dyn}\,(200\,{\rm GeV})\;\frac{\sqrt{\frac{1}{\langle K\rangle}+\frac{1}{\langle\pi\rangle}}\Big|_{\sqrt{s}}}{\sqrt{\frac{1}{\langle K\rangle}+\frac{1}{\langle\pi\rangle}}\Big|_{200\,{\rm GeV}}}$$

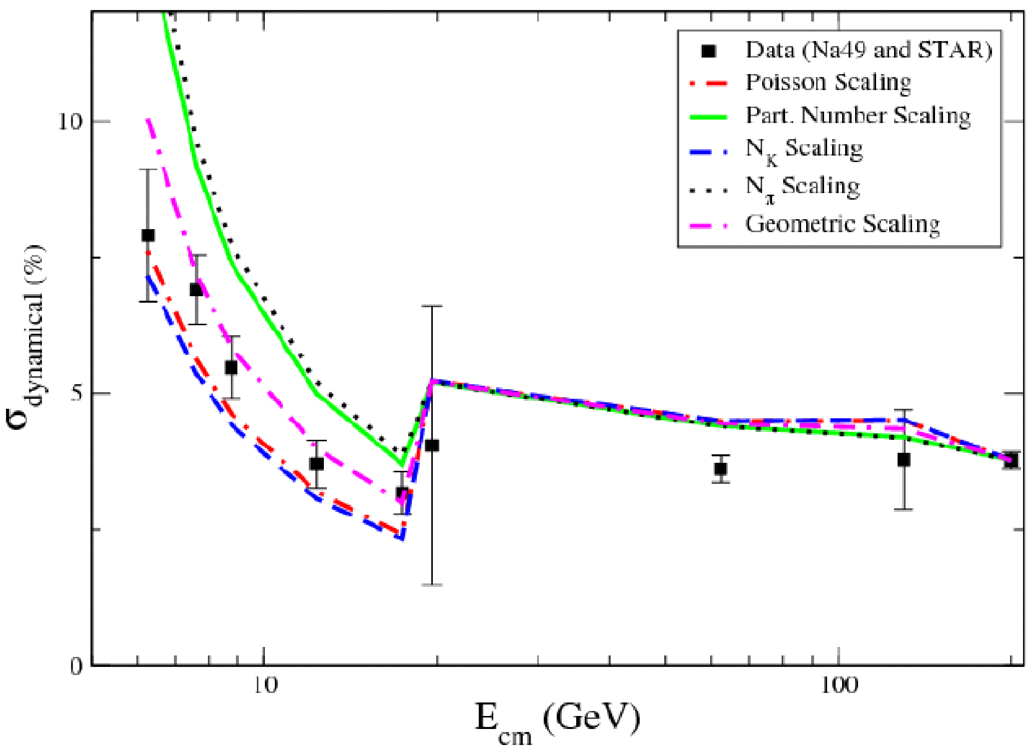
Part. Num. scaling:
$$\sigma_{\rm dyn}\left(\sqrt{s}\right) = \sigma_{\rm dyn}\,(200\,{\rm GeV})\;\frac{\sqrt{\langle K\rangle+\langle\pi\rangle}\Big|_{200\,{\rm GeV}}}{\sqrt{\langle K\rangle+\langle\pi\rangle}\Big|_{\sqrt{s}}}$$

Kaon Num. scaling:
$$\sigma_{\rm dyn}\left(\sqrt{s}\right) = \sigma_{\rm dyn}\,(200\,{\rm GeV})\;\frac{\sqrt{\langle K\rangle}\Big|_{200\,{\rm GeV}}}{\sqrt{\langle K\rangle}\Big|_{\sqrt{s}}}$$

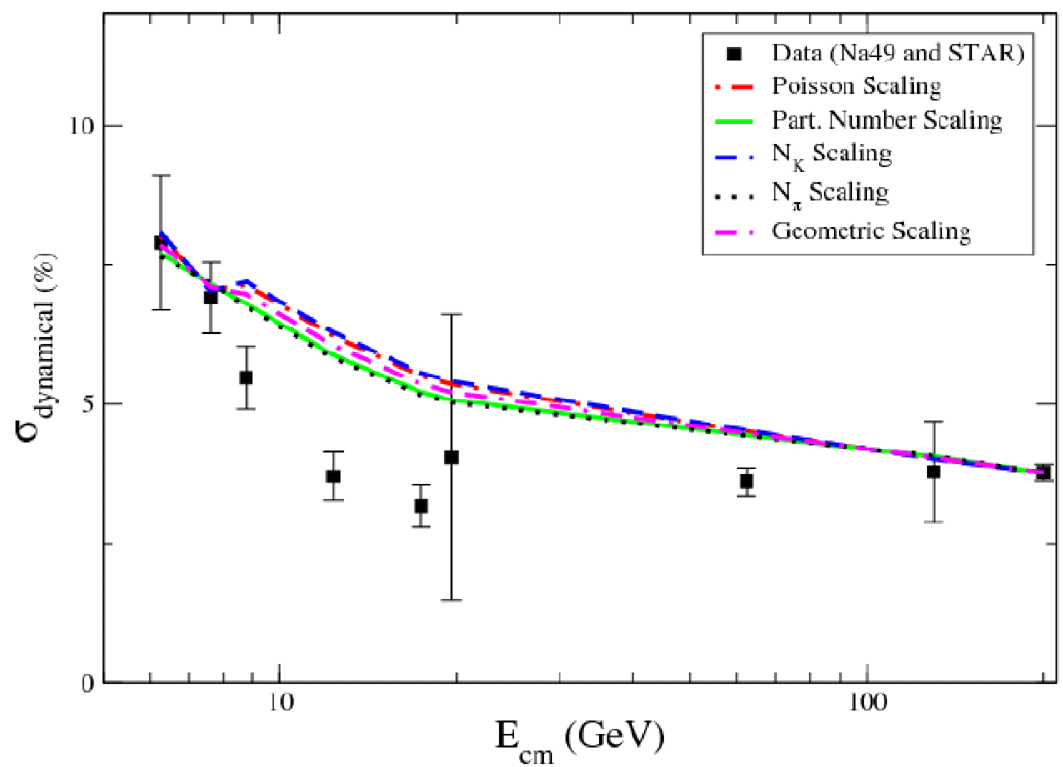
Pion Num. scaling:
$$\sigma_{\rm dyn}\left(\sqrt{s}\right) = \sigma_{\rm dyn}\,(200\,{\rm GeV})\;\frac{\sqrt{\langle\pi\rangle}\Big|_{200\,{\rm GeV}}}{\sqrt{\langle\pi\rangle}\Big|_{\sqrt{s}}}$$

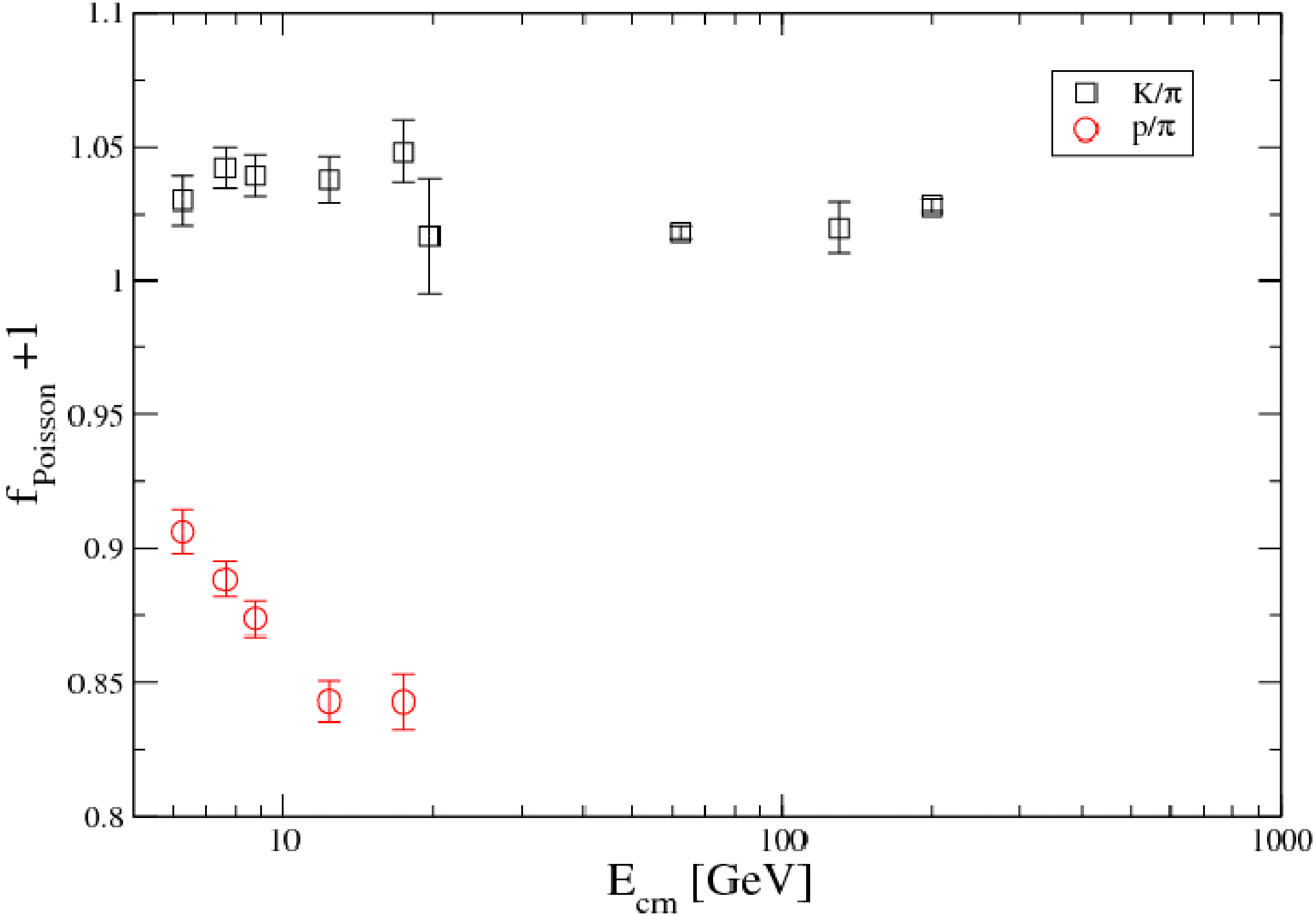
Geometric scaling:
$$\sigma_{\rm dyn}\left(\sqrt{s}\right) = \sigma_{\rm dyn}\,(200\,{\rm GeV})\;\frac{\left(\langle K\rangle\,\langle\pi\rangle\right)^{1/4}\Big|_{200\,{\rm GeV}}}{\left(\langle K\rangle\,\langle\pi\rangle\right)^{1/4}\Big|_{\sqrt{s}}}$$

Scaled with accepted Particles



Scaled with dN/dy

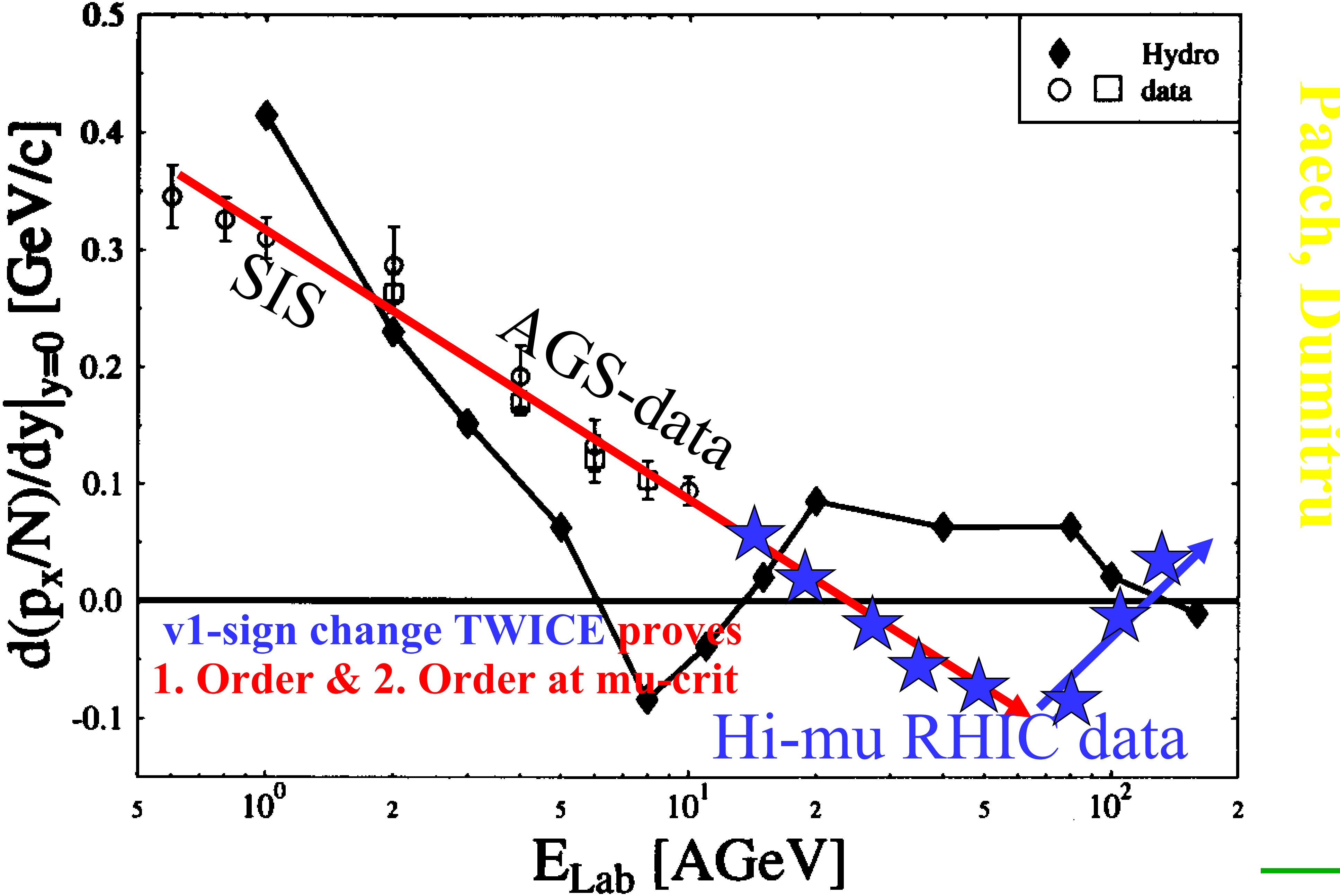




$$f_{Poisson} = \frac{\sigma^2_{dyn}}{\sigma^2_{Poisson}}$$

Other (“indirect”) observables

- Flow measurements (EOS, viscosity?)
- Lepton pairs? Only in conjunction with something else, such as baryon number fluctuations
  - Correlate baryon number with lepton yield in order to get after density fluctuations


Paech, Dumitru
Hydro
data
SIS
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v1-sign change TWICE proves
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Hi-mu RHIC data
$d(p_x/N)/dy|_{y=0}$ [GeV/c]
E_{Lab} [AGeV]
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
5
10^0
2
5
10^1
2
5
10^2
2

Critical Point and viscosities

CP is in universality class of liquid gas (Son, Stephanov)

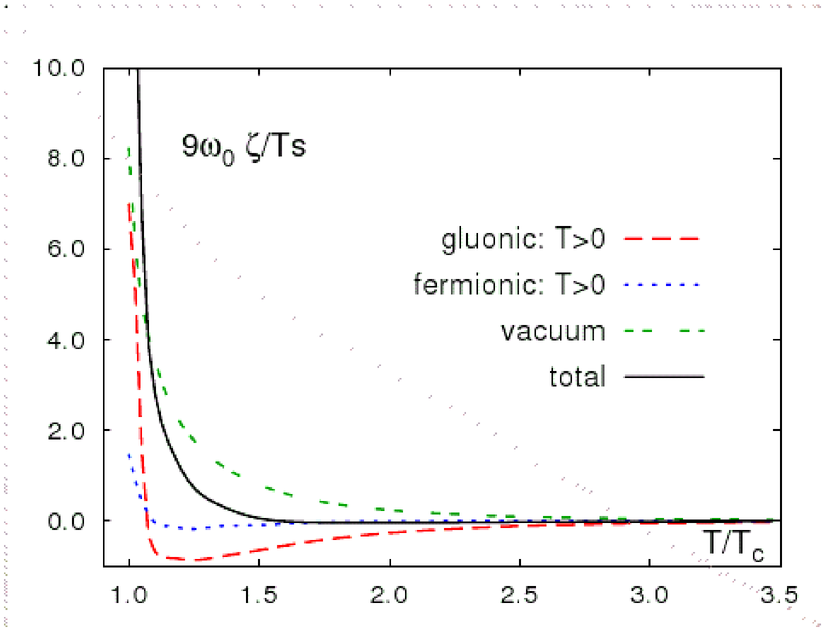
Hohenberg - Halperin Model H (Rev. Mod. Phys 49 (1977)):

$$\eta \sim \xi^{0.065}, \qquad \xi = \text{Correlation Length}$$

Shear viscosity diverges at CP

Bulk viscosity also diverges:
(Kharzeev, Turchin, Karsch arXiv:0711.0914)

Note: even large increase without PT
due to vacuum contribution



QCD critical point

- Order parameter: baryon density or scalar density
  - Actually it is a superposition
- Both scalar (chiral) an quark number susceptibilities diverge
- **Screening** (“space like”) masses vanish (“omega”, “sigma”)
  - not accessible by (time-like) dileptons
- Is it related to chiral transition at m_q=0 ?
- The transition is in same universality class as liquid gas! (Son, Stephanov)
  - Fluctuations are driven by density fluctuations; chiral field is just tagging
- CP “just” the end of of 1^{st} Order transiition
  - Spinodal instabilities

Observables for CP and co-existence

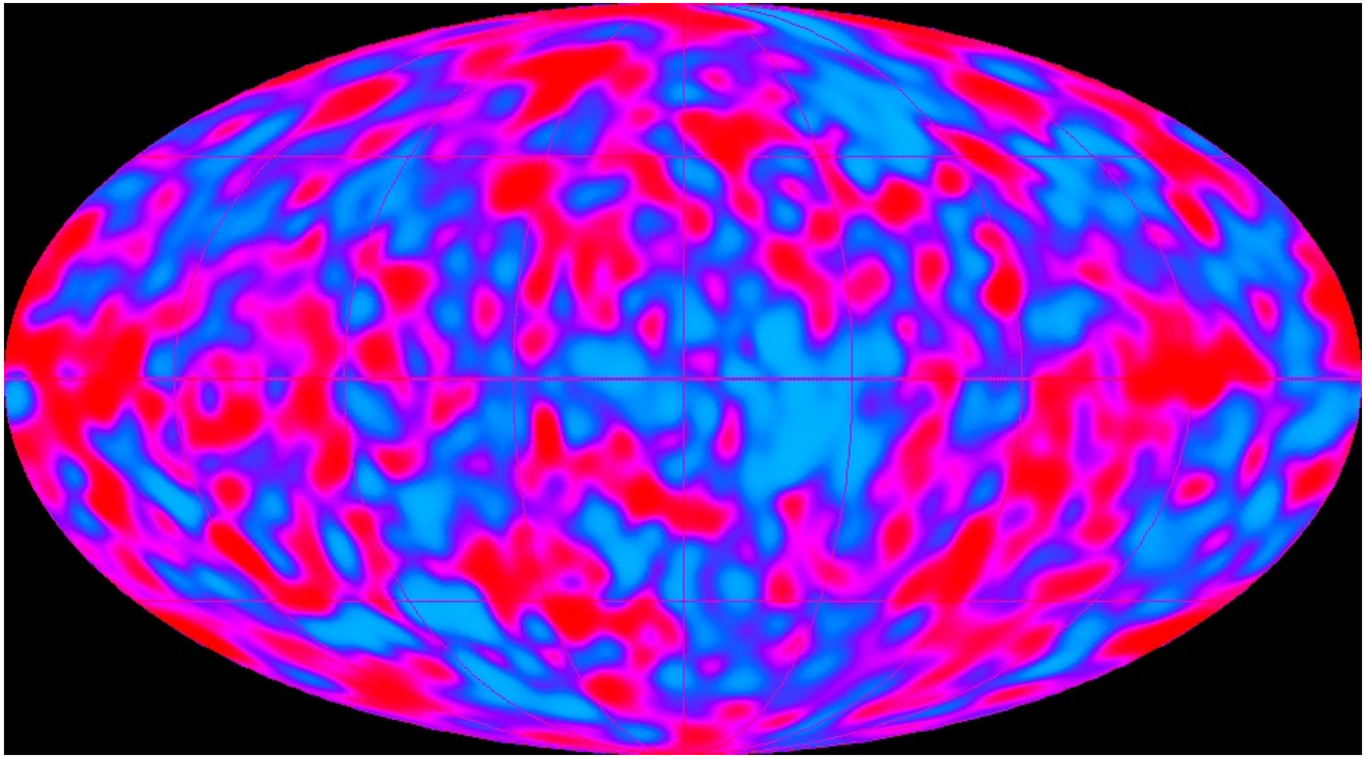
- Fluctuations (probably not of conserved charges)
- Correlations (spiondal blobs)
- Energy scan
- System size dependence (finite volume scaling ???)
  - centrality may not do
- Be prepared to measure everything
  - not clear (yet?) which observable couples strongest to baryon density
  - Should see effect in more than one observable
- So far NOTHING seen

“Little” Homework Problem

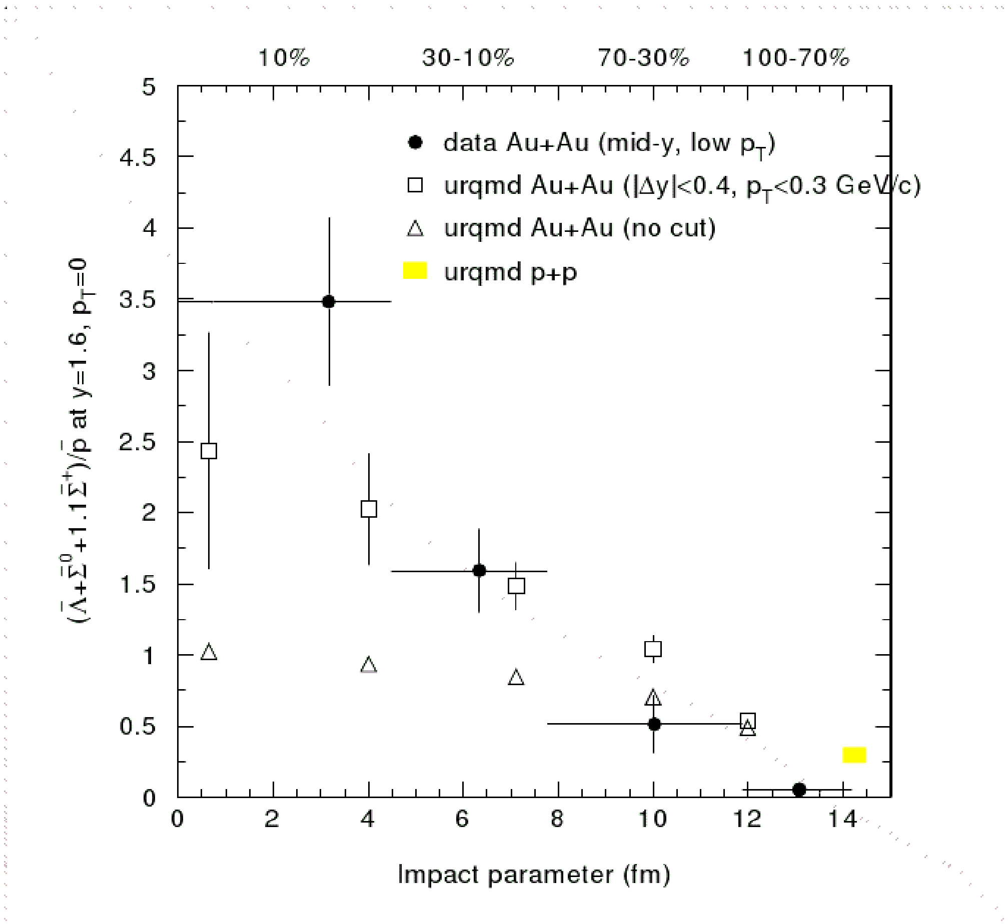
- Lattice find cross-over at $\mu=0$
  - Aoki et al, Nature 443:675-678,2006
- Can we rule out a first order transition from the RHIC data?

Summary

- Sign of phase co-existence CAN be seen in these type of experiments (Liquid Gas)
- Situation for QCD PT rather unsatisfactory
  - No firm theoretical guidance (Not even qualitative!)
  - Not clear how the phases present themselves (What are the “droplets”?
  - So far no evidence for or against PT of whatever kind



UrQMD and Lambda-bar / p-bar



F. Wang
nucl-ex/0010002

Strong enhancement mostly an effect of acceptance cut !?

Relativistic fluid dynamics

Energy-momentum tensor: $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu}$ $p(\varepsilon)$

Equation of motion: $\partial_\mu T^{\mu\nu} = 0$ $u^\mu = (\gamma, \gamma \mathbf{v})$

Small disturbance in a uniform stationary fluid $\varepsilon(x,t) = \varepsilon_0 + \delta\varepsilon(x,t)\ ,\ \delta\varepsilon \ll \varepsilon_0$

First order in $\delta\varepsilon$ *:*

$$\partial_t \delta\varepsilon(x,t) \approx (\varepsilon_0 + p_0)\partial_x v_x(x,t) \qquad p_0 \equiv p(\varepsilon_0)$$

$$(\varepsilon_0 + p_0)\partial_t v_x(x,t) \approx \partial_x p(x,t) \approx \frac{\partial p_0}{\partial \varepsilon_0}\partial_x \delta\varepsilon(x,t)$$

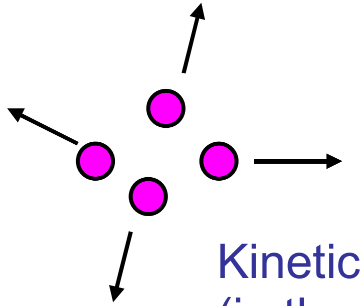
Sound waves:

$$\boxed{\partial_t^2 \delta\varepsilon(x,t) = \frac{\partial p_0}{\partial \varepsilon_0}\partial_x^2 \delta\varepsilon(x,t)} \qquad v_s^2 = \frac{\partial p}{\partial \epsilon}$$

J. Randrup

Kinematic clumping =>

Invariant-mass correlations



Total four-momentum:

$$P\{\boldsymbol{p}_n\} \;=\; \sum_n (E_n, \boldsymbol{p}_n)$$

Kinetic energy per particle (in the *N*-body CM frame):

$$\kappa_N\{\boldsymbol{p}_n\} \;=\; \frac{1}{N}\left[[P\{\boldsymbol{p}_n\} \cdot P\{\boldsymbol{p}_n\}]^{\frac{1}{2}} \;-\; \sum_n m_n \right]$$

Distribution of κ :

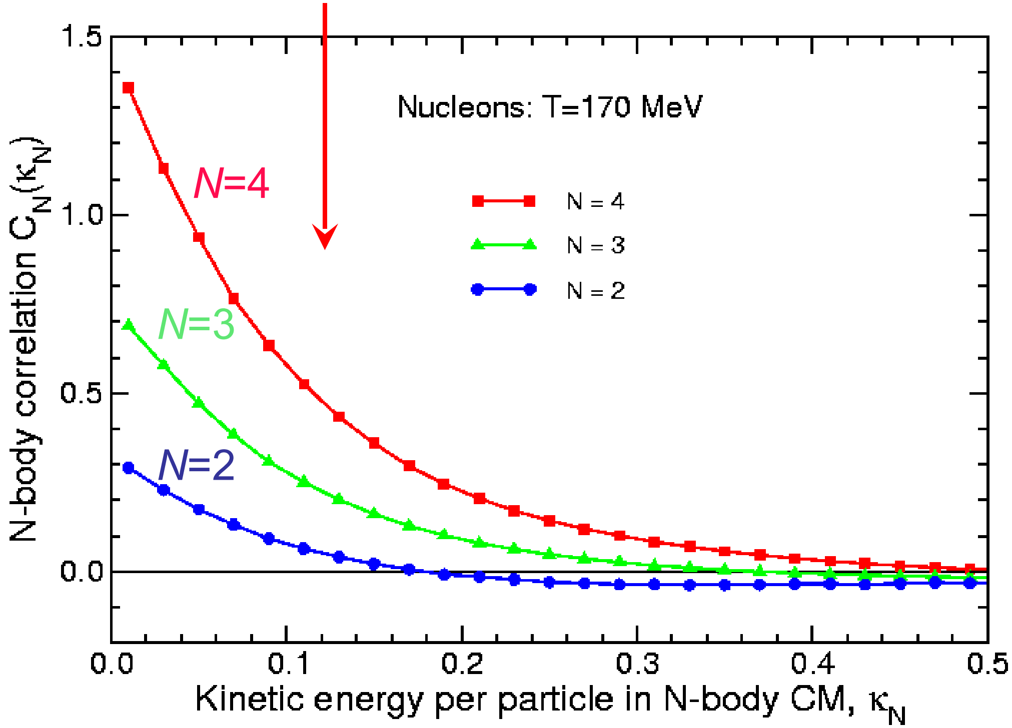
$$P_N(\kappa) \;\equiv\; \prec \delta(\kappa - \kappa_N\{\boldsymbol{p}_n\}) \succ$$

Correlation function:

$$C_N(\kappa) \;\equiv\; P_N(\kappa) \;/\; P_N^0(\kappa) \;-\; 1$$

Same event / *Mixed events*

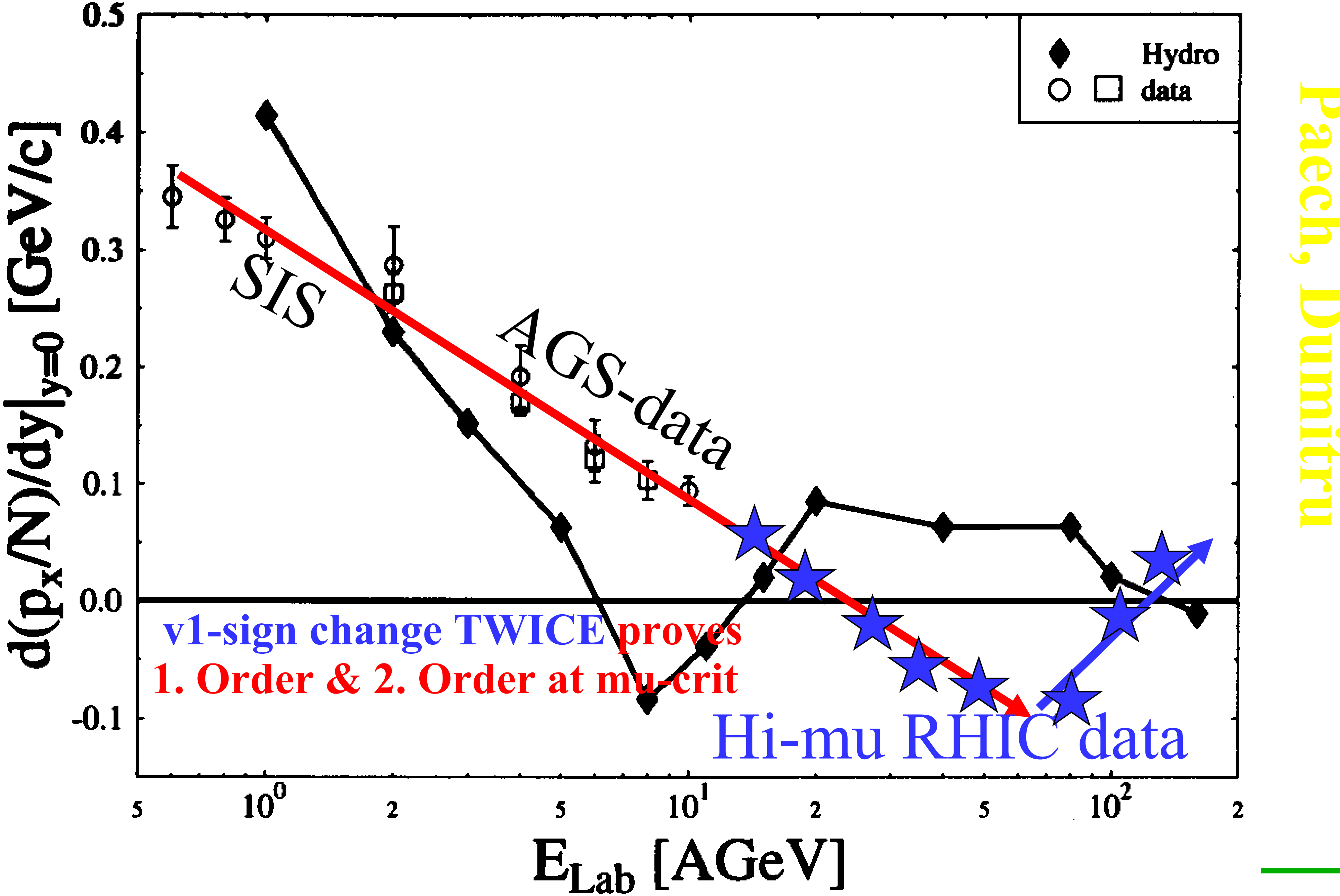
- is *enhanced* at $\kappa \approx T$



Higher-order correlations stand out more clearly!

(but require larger samples)

[J. Randrup, J. Heavy Ion Physics 22 (2005) 69]



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